Journal of Optics B: Quantum and Semiclassical Optics

Related content

- Preparing highly excited Fock states of a cavity field using driven atoms
A T Avelar and B Baseia
- Sculpturing coherent states to get highly excited Fock states for stationary and travellingfields
L P A Maia, B Baseia, A T Avelar et al.
- Nonprobabilistic teleportation of a field state via cavity QED
C R Carvalho, E S Guerra, Ginette Jalbert et al.

Generation of superposed phase states via Raman interaction

To cite this article: A T Avelar *et al* 2004 *J. Opt. B: Quantum Semiclass. Opt.* **6** 383

Recent citations

- Dynamics of nonclassical correlations via local quantum uncertainty for atom and field interacting into a lossy cavity QED
J.S. Sales *et al*
- QUANTUM STATE TRANSFER AND HADAMARD GATE FOR COHERENT STATES
THIAGO PRUDÊNCIO
- Coherent manipulations of atoms using laser light
Bruce Shore

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF OPTICS B: QUANTUM AND SEMICLASSICAL OPTICS

J. Opt. B: Quantum Semiclass. Opt. **6** (2004) 383–386 PII: S1464-4266(04)79476-8

Generation of superposed phase states via Raman interaction

A T Avelar[1,3], L A de Souza[1], T M da Rocha Filho[1] and B Baseia[2]

[1] Instituto de Física, Universidade de Brasília, 70.919-970, Brasília (DF), Brazil
[2] Instituto de Física, Universidade Federal de Goiás, 74.001-970, Goiânia (GO), Brazil

E-mail: ardiley@unb.br (A T Avelar)

Received 16 April 2004, accepted for publication 2 August 2004
Published 23 August 2004
Online at stacks.iop.org/JOptB/6/383
doi:10.1088/1464-4266/6/10/001

Abstract
A recent proposal producing superpositions of two truncated phase states in a high-Q microwave cavity has been presented. The scheme was based on dispersive interactions of two-level Rydberg atoms with a single mode field inside a cavity. Here we consider a simplified scheme doing the same, via Raman interaction of the field with degenerate three-level atoms. The extension to 2^K phase states components ($K > 1$) is discussed.

Keywords: quantum state engineering, Raman interaction, Pegg–Barnett phase state

1. Introduction

An important topic in quantum optics is one concerned with tailoring interesting nonclassical states of the quantized electromagnetic field, either for stationary modes trapped inside high-Q cavities [1–5] or for travelling modes [6–13]. Besides the number and the coherent state, another basic state of the quantized electromagnetic field is the phase state. It plays the role of complementing the number state, since the number operator $\hat{N}$ and the phase operator $\hat{\phi}$ constitute a conjugate pair of observables [14]: $[\hat{\phi}, \hat{N}] = i$. This commutation relation was questioned for a long time and the nature of the phase of a quantized electromagnetic field has remained an enigma, also due to the lack of a corresponding Hermitian phase operator [15]. Such difficulties placed the phase state in the unique position of being a classical observable with no corresponding Hermitian operator counterpart. In practice this situation was not so uncomfortable since most experiments then involved vacuum and thermal states—for which the phase has no relevance. However, the advent of laser light [16] and squeezed light [17] has renewed interest in the problem. Despite being not consensual [18], the problem of the phase state (and Hermitian phase operator) was well addressed in 1988, by Pegg–Barnett [19]. Generation of the phase state has been studied recently, for stationary and travelling fields [8, 9, 20].

More recently yet, a proposal generating *superpositions of truncated phase states* (STPS) has also been presented for fields trapped inside a high-Q cavity [5] and for travelling fields [6]. In [5] the scheme relies on (nonunitary) collapse of an entangled state caused by a process of selective atomic detection. The mentioned entangled state describes two interacting sub-systems, namely a two-level (Rydberg) atom and a single-mode field. The scheme is similar to others related to the generation of the so-called 'Schrödinger cat state' [4].

In the present paper we will consider an alternative and simpler scheme generating the mentioned superposed states in cavity-QED. We will employ a different Hamiltonian, the physical system so modelled having a Raman interaction of degenerate Λ-type three-level atoms with a single mode cavity field. This procedure, which is inspired by [21], simplifies the traditional procedure in [4] by ignoring the use of Ramsey zones. Actually, our approach goes beyond [21] by economizing two Ramsey zones instead of one.

This paper is arranged as follows: in section 2, for comparison and completeness, we present a summary of the previous scheme used in [5]. Section 3 discusses our simplified scheme and section 4 contains comments and conclusions.

2. Generation of STPS via dispersive interaction

To create a stationary field inside a microwave cavity in a STPS we start from a single-mode truncated phase state (TPS) previously prepared in it. The method requires the use of a

[3] Author to whom any correspondence should be addressed.

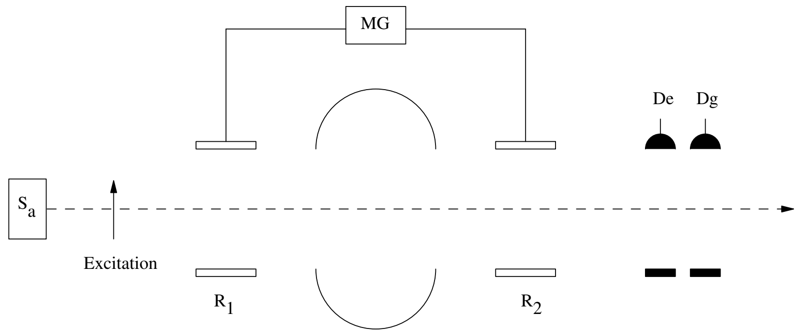

Figure 1. Schematic illustration of the setup creating the STPS in a high-Q microwave cavity using dispersive atomic probes.

high-Q superconducting cavity C, placed between two low-Q cavities (Ramsey zones R_1 and R_2), as shown in figure 1.

A two level Rydberg atom is ejected with selected velocity from a source S_a and prepared in circular excited state $|e\rangle$ (principal quantum number $n = 51$ in the case of rubidium) by an appropriated laser beam. When the atom crosses both Ramsey zones R_1 and R_2 it interacts with classical fields produced by a microwave generator (MG). These interactions are resonant with the atomic transition between the states $|e\rangle$ and $|g\rangle$ ($n = 50$), the field intensities being adjusted to produce a $\pi/2$ rotation in the atomic space, namely,

$$|e\rangle \longrightarrow (|g\rangle + |e\rangle)/\sqrt{2}, \tag{1}$$

$$|g\rangle \longrightarrow (|g\rangle - |e\rangle)/\sqrt{2}. \tag{2}$$

An auxiliary (third) atomic level $|i\rangle$ ($n = 52$) is crucial in the scheme: the cavity frequency is adjusted close to resonance (detuned by small δ) with the transition $|e\rangle \longrightarrow |i\rangle$, but far from the transition $|g\rangle \longrightarrow |e\rangle$, as shown in figure 2. This avoids the change of number of photons in the field, hence only the phase may vary; this characterizes the atom–field interaction as dispersive, instead of resonant. This interaction is described by the effective atom–field Hamiltonian [22],

$$H_{\mathrm{int}} = \hbar\omega_{\mathrm{eff}}\hat{a}^{\dagger}\hat{a}(|i\rangle\langle i| - |e\rangle\langle e|), \tag{3}$$

where $\omega_{\mathrm{eff}} = 2d^2/\delta$ and d is the atomic dipole moment. So, the atom produces a phase shift in the field state when it crosses the cavity in its excited state $|e\rangle$, but no phase shift results when the atom is in the ground state $|g\rangle$.

Next, consider the field inside the cavity initially prepared in a TPS, as shown in [8, 20], given by [19]

$$|\theta_l\rangle = \frac{1}{\sqrt{N+1}}\sum_{n=0}^{N} \mathrm{e}^{\mathrm{i}n\theta_l}|n\rangle, \tag{4}$$

where

$$\theta_l = \theta_0 + l\left(\frac{2\pi}{N+1}\right), \tag{5}$$

where $l = 0, 1, 2, \ldots, N$ and θ_0 is a reference phase. The evolution of the *entangled* atom–field state, as the atom crosses the system, follows the steps shown in table 1 (up to normalization and using $\mathrm{e}^{\mathrm{i}\phi\hat{a}^{\dagger}\hat{a}}|\theta_l\rangle = |\theta_l + \phi\rangle$, with $\phi = \omega_{\mathrm{eff}}t$; t stands for the time spent by the atom crossing the cavity).

Thus, if the atom is detected in the state $|g\rangle$ ($|e\rangle$), the field in the cavity is projected onto the state $|\theta_m\rangle + |\theta_l\rangle$ ($|\theta_m\rangle - |\theta_l\rangle$),

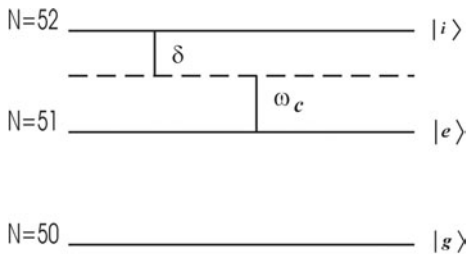

Figure 2. Schematic diagram of the two-level atom interacting with the single-mode field.

Table 1. Evolution of the atom–field state as the atom crosses the system.

Atom position	$\vert\Psi\rangle_{\text{atom+field}}$
Before R_1	$\vert e\rangle\vert\theta_l\rangle$
After R_1	$(\vert g\rangle + \vert e\rangle)\vert\theta_l\rangle$
After C	$\vert g\rangle\vert\theta_l\rangle + \vert e\rangle\vert\theta_l + \phi\rangle$
After R_2	$\vert g\rangle(\vert\theta_l + \phi\rangle + \vert\theta_l\rangle) + \vert e\rangle(\vert\theta_l + \phi\rangle - \vert\theta_l\rangle)$

where $\theta_m = \phi + \theta_l$. Note that the state $|\theta_m\rangle \pm |\theta_l\rangle$ will correspond to the wanted STPS (up to normalization), if the phase ϕ satisfies the condition [19]

$$\phi = \theta_m - \theta_l = (m - l)\left(\frac{2\pi}{N+1}\right), \tag{6}$$

with $m + 1 \leqslant l \leqslant m + N$.

3. Generation of STPS via Raman interaction

Figure 3 displays our simplified setup preparing the STPS inside a high-Q cavity: S_a represents ‘source of atoms’; ‘excitation’ prepares the (highly excited) Rydberg atom; C represents the cavity; D_e and D_g stand for ‘atomic detectors’.

In this scheme a Λ-type three-level atom is injected into the cavity, the lower energies E_g and E_e (corresponding to the states $|g\rangle$ and $|e\rangle$) being assumed as equal [21], as shown in figure 4. When the atomic transition frequency ω_0 is highly detuned from the cavity frequency ω_{f}, i.e. $\Delta = \omega_0 - \omega_{\mathrm{f}}$ being large, the upper state $|i\rangle$ can be adiabatically eliminated [23]. Under this condition the effective Hamiltonian describing such a system reads [24]

$$H_{\mathrm{e}} = -\beta a^{\dagger}a(|e\rangle\langle g| + |g\rangle\langle e|) - a^{\dagger}a(\beta_1|g\rangle\langle g| + \beta_2|e\rangle\langle e|), \tag{7}$$

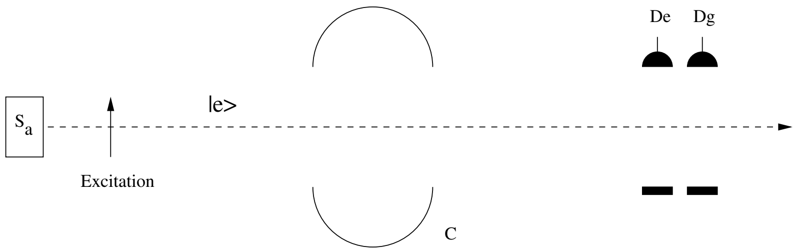

Figure 3. Scheme of the setup creating the STPS inside a high-Q microwave cavity.

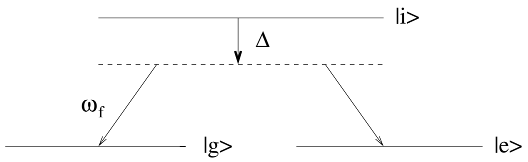

Figure 4. Schematic diagram of the degenerate Λ-type three-level atom interacting with the single-mode field.

where $a^\dagger(a)$ is the creation (annihilation) operator for the cavity mode, and

$$\beta = \frac{\lambda_1\lambda_2}{\Delta}, \qquad \beta_1 = \frac{\lambda_1^2}{\Delta}, \qquad \beta_2 = \frac{\lambda_2^2}{\Delta},$$

with $\lambda_1(\lambda_2)$ denoting the coupling constant between the transition $|e\rangle \to |i\rangle$ ($|g\rangle \to |i\rangle$) and the cavity mode. At this point, the assumption $\lambda_1 = \lambda_2 = \lambda$ (hence $\beta_1 = \beta_2 = \beta$) is a natural simplification.

The atom crossing the cavity is assumed to be initially prepared in the excited state $|e\rangle$ and interacts with an initial TPS prepared inside the cavity. Thus, since the Hamiltonian (7) is time-independent, the evolution operator is given by $U(t) = \exp(-\mathrm{i}tH/\hbar)$ and its application to the initial state describing the whole atom–field system, $|\Psi_{\mathrm{AF}}(0) = |e\rangle|\theta_l\rangle$, results in

$$|\Psi_{\mathrm{AF}}(t)\rangle = \tfrac{1}{2}\left\{\left[\mathrm{e}^{2\mathrm{i}\hat{n}\beta t} - 1\right]|g,\theta_l\rangle + \left[\mathrm{e}^{2\mathrm{i}\hat{n}\beta t} + 1\right]\right\}|e,\theta_l\rangle. \quad (8)$$

As a consequence, if one detects the atom in the state $|e\rangle$ ($|g\rangle$) then the field, in the number-basis representation, collapses into the state $|\Psi^{(+)}(\tau)\rangle(|\Psi^{(-)}(\tau)\rangle)$ as follows

$$|\Psi_{\mathrm{F}}^{(\pm)}(\tau)\rangle = \mathcal{N}^{(\pm)} \sum_{n=0}^{N} \left[\mathrm{e}^{\mathrm{i}n(\phi(\tau)+\theta_l)} \pm \mathrm{e}^{\mathrm{i}n\theta_l}\right]|n\rangle, \quad (9)$$

where $\phi(\tau) = 2\beta\tau$, with $\tau = L/v_{\mathrm{a}}$, L and v_{a} standing for the cavity lengthy and atomic velocity, respectively; $\mathcal{N}^{(\pm)}$ stands for normalization, and has no relevance for the present purpose. Note that the state $|\Psi_{\mathrm{F}}^{(\pm)}(\tau)\rangle$ in the equation (9) will correspond to the wanted STPS (up to normalization),

$$|\Psi_{\mathrm{F}}^{(\pm)}(\tau)\rangle = |\theta_m\rangle \pm |\theta_l\rangle, \quad (10)$$

where $\theta_m = \phi(\tau) + \theta_l$ and the phase $\phi(\tau)$ satisfying the prescription given by equation (6). Accordingly, the phase $\phi(\tau)$ is controlled by the time τ spent by atom to cross the cavity. The foregoing scheme requires the existence of the truncated phase state $|\theta_l\rangle$, previously prepared inside the cavity, a point discussed in [8, 9, 20].

At this point, to be specific, consider the generation of the STPS $|\theta_l\rangle + |\theta_m\rangle$ of dimension $N = 7$, with $l = 0$ and $m = 1$. We take typical values for the parameters involved [25]: the coupling constant $\lambda \simeq 7 \times 10^5$ s^{-1} and the detuning $\Delta/2\pi \simeq 39$ MHz. These data lead to $\beta = \lambda^2/\Delta \simeq 2 \times 10^4$ s^{-1} and the interaction time $\tau = \pi(m-l)/(N+1)\beta \simeq 1.9 \times 10^{-5}$ s. This result requires the Rydberg atom with velocity $v \simeq 382$ m s^{-1}, belonging to typical intervals available in laboratories, $v \sim 300$–500 m s^{-1} [25]. So the scheme is experimentally feasible within the realm of microwave. It simplifies the previous scheme implemented in [5], also concerned with *two* phase states components.

Now, how to go further, preparing such superposition with 2^K components? The extension to this case is achieved by assuming K atoms passing throughout the system one finds (with success probability about $1/2^K$) the generalized superposition given by

$$|\Psi_K(\phi)\rangle = \mathcal{N}(\phi) \sum_{j=0}^{J_K} (\mathrm{e}^{\mathrm{i}\phi\hat{n}}|\theta_j\rangle + |\theta_j\rangle), \quad (11)$$

where $|\theta_j\rangle = \mathrm{e}^{\frac{\mathrm{i}j\phi\hat{n}}{2^{K-1}}}|\theta_0\rangle$, $J_K = 2^{K-1} - 1$ and $\mathcal{N}(\phi)$ standing for normalization of the state. The extended procedure works in a similar way as the extension studied in [26] for the generation of 2^K circular coherent states in the phase space. Here the novelty is the necessary inclusion of the prescription given in equation (6), which specifies the (Pegg–Barnett [19]) phase character of the state.

In practice, the extended case is accompanied by some difficulties, such as the diminution of both probability and fidelity of generation [4, 27], including the problem of decoherence of a state being prepared inside a cavity [28] due to unavoidable interaction of the system with its environment. With respect to the nonclassical properties exhibited by a state, those for superpositions involving two components of phase states have already been studied in [5]; since the properties of a state do not depend on the way the state has been generated, then their study here would be redundant. However, since the extended superposition of 2^K coherent states was shown to exhibit remarkable nonclassical properties [26, 29], then the detailed study of properties of the extended superposition of 2^K phase states deserves further attention.

4. Comments and conclusion

In this paper we have discussed a model-Hamiltonian describing the Raman interaction of a Λ-type three-level atom with a single-mode field. This allows one to generate a STPS starting from a TPS previously prepared inside a high-Q microwave cavity. The procedure is simplified, since it economizes the two Ramsey zones used in [5]. Although we have been inspired by [21], who economize the traditional second Ramsey zone, the present approach goes one step beyond by using no Ramsey zone. This improvement in comparison with [21] comes from assuming that the atom enters the cavity in the excited state $|e\rangle$, which makes the first Ramsey zone also useless; otherwise the first Ramsey zone would become necessary. Since the time intervals spent by atoms crossing the cavity and crossing a Ramsey zone are of the same order, about 1 μs [30], then the present simplifications have significative importance, by economizing the arrangement and the time of experiment. Actually, all schemes for stationary fields require the use of cavities having high quality to sustain the prepared field state for sufficiently 'large' times, in view of its degradation due to decoherence of the state caused by unavoidable field–environment interactions [28].

The extension of the present STPS having two components, for the case of 2^K components (K stands for the number of atoms, crossing the cavity one-by-one), has been briefly discussed (end of section 3). A detailed treatment of this generalization, especially concerning the nonclassical properties exhibited by these kinds of state emerging from different values of the parameter K and different atomic state detections ($|e\rangle$ or $|g\rangle$), is under investigation and will be published elsewhere.

Acknowledgments

We thank the CNPq (ATA, TMRF, BB) and CAPES (LAS), Brazilian agencies, for partial support.

References

[1] Vogel K, Akulin V M and Schleich W P 1993 *Phys. Rev. Lett.* **71** 1816
Parkins A S *et al* 1996 *Phys. Rev. Lett.* **71** 3095
[2] Brune M *et al* 1994 *Phys. Rev. Lett.* **72** 3339
[3] Davidovich L *et al* 1994 *Phys. Rev.* A **50** R895
[4] Brune M *et al* 1990 *Phys. Rev. Lett.* **65** 976
Brune M *et al* 1992 *Phys. Rev.* A **45** 5193
[5] de Souza L A, Baseia B and Malbouisson J M C 2003 *Phys. Lett.* A **309** 5
[6] Avelar A T, Escher B M and Baseia B 2003 *Phys. Lett.* A **318** 161
[7] Pegg D T, Phillips L S and Barnett S M 1998 *Phys. Rev. Lett.* **81** 1604
[8] Guimarães Y A *et al* 2000 *Phys. Lett.* A **268** 260
[9] Villas-Boas C J *et al* 2001 *Phys. Rev.* A **63** 55801
[10] Yurke B and Stoler D 1986 *Phys. Rev. Lett.* **57** 13
[11] Dakna M *et al* 1999 *Phys. Rev.* A **59** 1658
Dakna M *et al* 1999 *Phys. Rev.* A **60** 726
[12] Valverde C *et al* 2003 *Phys. Lett.* A **315** 213
Avelar A T, de Almeida N G and Baseia B 2003 *J. Opt. B: Quantum Semiclass. Opt.* **5** 1
[13] Gerry C C 1999 *Phys. Rev.* A **59** 4095
D'Ariano G M *et al* 2000 *Phys. Rev.* A **61** 053817
[14] Dirac P A M 1927 *Proc. R. Soc.* A **114** 243
[15] Louisell W H 1963 *Phys. Lett.* **7** 60
Susskind L and Glogower J 1964 *Physics* **1** 49
Carruthers P and Nieto M M 1968 *Rev. Mod. Phys.* **40** 411
Lynch R 1995 *Phys. Rep.* **256** 367
[16] Maimam T H 1960 *Nature* **187** 493
Lamb W E 1964 *Phys. Rev.* A **134** 1429
Haken H 1984 *Laser Theory* (Berlin: Springer)
[17] Slusher R E *et al* 1985 *Phys. Rev. Lett.* **55** 2409
[18] Torgerson J R and Mandel L 1996 *Phys. Rev. Lett.* **76** 3939
Ma X and Rhodes W 1991 *Phys. Rev.* A **43** 2576
Vorontsov Yu I and Rembovsky Yu A 1999 *Phys. Lett.* A **254** 7
Vorontsov Yu I and Rembovsky Yu A 1999 *Phys. Lett.* A **262** 486
See also: Vaccaro J A, Pegg D T and Barnett S M 1999 *Phys. Lett.* A **262** 483
[19] Pegg D T and Barnett S M 1988 *Europhys. Lett.* **6** 483
Pegg D T and Barnett S M 1989 *Phys. Rev.* A **39** 1665
Pegg D T and Barnett S M 1989 *J. Mod. Opt.* **36** 7
[20] Aragão A, Avelar A T and Baseia B 2004 States of the quantized electromagnetic field with highly concentrated phase distribution, at press
[21] Li H and Wu L 2001 *Opt. Commun.* **197** 97
[22] Gerry C C and Knight P L 1997 *Am. J. Phys.* **65** 964
[23] Steinbach J, Twamley J and Knight P L 1997 *Phys. Rev.* A **56** 4815
[24] Guo G C and Zheng S B 1996 *Phys. Lett.* A **223** 332
[25] Brune M, Raimond J M and Haroche S 1987 *Phys. Rev.* A **35** 154
[26] Malbouisson J M C and Baseia B 1999 *J. Mod. Opt.* **46** 2015
Maia L P A *et al* 2004 *J. Opt. B: Quantum Semiclass. Opt.* **6** 351
[27] Lutterbach L G and Davidovich L 2000 *Phys. Rev.* A **61** 023813
[28] Haroche S 1992 *Fundamental Systems in Quantum Optics* (*Les Houches (1990)*) ed J Dalibard *et al* (New York: Elsevier Science)
Brune M *et al* 1996 *Phys. Rev. Lett.* **77** 4887 see also [22]
[29] Borges Paula M *et al* 2004 Sculpturing squeezed states to get highly excited number states, at press
Aragão A *et al* 2004 *Phys. Lett.* A **329** 284
Borges Paula M *et al* 2004 Generation and properties of 2^n superposed squeezed states, at press
[30] Osnaghi S *et al* 2001 *Phys. Rev. Lett.* **79** 037902