Journal of Optics B: Quantum and Semiclassical Optics

Preparing highly excited Fock states of a cavity field using driven atoms

To cite this article: A T Avelar and B Baseia 2005 *J. Opt. B: Quantum Semiclass. Opt.* **7** 198

View the article online for updates and enhancements.

Related content

- Sculpturing coherent states to get highly excited Fock states for stationary and travellingfields
  L P A Maia, B Baseia, A T Avelar et al.
- Generation of superposed phase states via Raman interaction
  A T Avelar, L A de Souza, T M da Rocha Filho et al.
- Teleportation of an entangled atomic state using cavity-assisted collisions
  Muhammad Irfan, Mudassar Aqueel Ahmad, Sajid Qamar et al.

Recent citations

- Dynamics of nonclassical correlations via local quantum uncertainty for atom and field interacting into a lossy cavity QED
  J.S. Sales *et al*
- Strong-Driving-Assisted Preparation of Superpositions of Two-Mode Coherent States in Cavity QED
  Wan-Jun Su and Jian-Min Huang
- Relation between fundamental estimation limit and stability in linear quantum systems with imperfect measurement
  Naoki Yamamoto

This content was downloaded from IP address 200.137.211.50 on 14/06/2018 at 13:30

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF OPTICS B: QUANTUM AND SEMICLASSICAL OPTICS
J. Opt. B: Quantum Semiclass. Opt. **7** (2005) 198–200 doi:10.1088/1464-4266/7/6/007

Preparing highly excited Fock states of a cavity field using driven atoms

A T Avelar[1,2,3] **and B Baseia**[2]

[1] Instituto de Física, Universidade de Brasília, 70.919-970, Brasília (DF), Brazil
[2] Instituto de Física, Universidade Federal de Goiás, 74.001-970, Goiânia (GO), Brazil

E-mail: ardiley@unb.br (A T Avelar)

Received 23 December 2004, accepted for publication 8 April 2005
Published 17 May 2005
Online at stacks.iop.org/JOptB/7/198

Abstract
Inspired by a previous work (Zheng and Guo 1998 *Phys. Lett.* A **244** 512) we propose a scheme for creating Fock states $|2^N\rangle$ of the quantized radiation field using atoms driven by a classical field. This scheme is an alternative to another published one (Maia *et al* 2004 *J. Opt. B: Quantum Semiclass. Opt.* **6** 351). Comparisons between them and others are discussed.

Keywords: quantum state engineering, Fock states

Generation of Fock states has potential relevance for various applications, such as secure quantum communication [1], quantum cryptography [2], optimal capacity coding in quantum channels [3], high-precision quantum interferometry [4]. However, the generation in laboratories is not trivial, mainly when concerned with highly excited Fock states. Recent experimental results for one-photon [5] and two-photon [6] Fock states have been achieved in high-Q cavities. From a theoretical point of view, it has been shown that Fock states can be prepared by superposing coherent states distributed on a circle in the phase space [7, 8]. This has motivated experimental schemes for generating certain kinds of such superpositions as states of the quantized electromagnetic field, for stationary modes trapped inside high-Q cavities [9, 10] and for travelling modes [10].

In a previous paper Zheng and Guo [11] proposed a cavity QED technique for generating arbitrary superpositions of coherent states on a circle in the phase space. The scheme relies on the interaction of a quantized cavity field with a highly excited (Rydberg) atom driven by a classical field. In the present report we propose convenient adaptations of this scheme which allow us to obtain a family of Fock states $|2^N\rangle$. This family is remarkable in the sense that it refers to a special set of highly excited Fock states: $|8\rangle$, $|16\rangle$, $|32\rangle$, ...

Figure 1 displays the set-up used for preparing Fock states inside a high-Q cavity: S_A represents the 'source of atoms', the 'excitation' prepares a highly excited (Rydberg) atom, 'C' represents the cavity, 'MG' stands for 'microwave generator', and D_e (D_g) stands for the atomic ionization detector for the state $|e\rangle$ ($|g\rangle$). The cavity is prepared at low temperature ($T \simeq 0.6$ K) in order to reduce the presence of thermal photons [16].

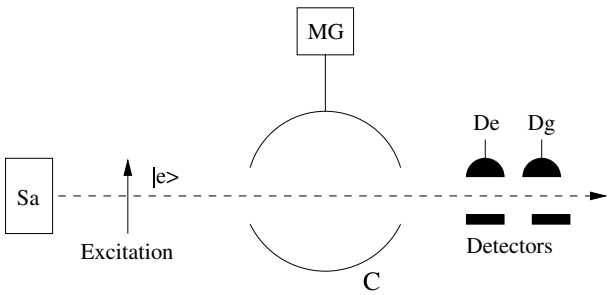

Figure 1. Scheme of the set-up for creating fields in Fock states.

The procedure in [11] employs the driven Jaynes–Cummings Hamiltonian [13] modelled as

$$\hat{H} = \omega_c \hat{a}^\dagger \hat{a} + \omega_a(t)\hat{S}_z + g(\hat{a}^\dagger \hat{S}^- + \hat{a}\hat{S}^+) + \varepsilon(t)e^{i\omega_L t}\hat{S}^+ + \varepsilon^*(t)e^{-i\omega_L t}\hat{S}^- \quad (1)$$

where $\hat{a}$ ($\hat{a}^\dagger$) is the photon annihilation (creation) operator, $\hat{S}^\pm$ and $\hat{S}_z$ stand for the electronic flip operators, $\omega_a(t)$ is the frequency of atomic transitions between the excited state $|e\rangle$ and the ground state $|g\rangle$, ω_c and ω_L are the frequencies of the cavity and classical fields, respectively; g is the atom–cavity coupling constant and $\varepsilon(t)$ is the amplitude of the classical field.

In [11] the parameters $\omega_a(t)$ and $\varepsilon(t)$ are time dependent and they are assumed as having external control. In the present

[3] Author to whom any correspondence should be addressed.

scenario we do not need to vary the amplitude of the classical field, and we will set $\varepsilon(t) = \varepsilon$, for convenience. This makes the control of interaction times easier than in [11]. To be pedagogical, we explain the time evolution of the system step by step, as follows:

First step: consider the atomic transition frequency initially resonant with the classical field and far off resonance with the quantized cavity field. Hence the Hamiltonian describing the atomic system acted upon by the classical field is

$$\hat{H}_{\rm driven} = \omega_{\rm L}\hat{S}_z + \varepsilon\hat{S}^{+}{\rm e}^{{\rm i}\omega_{\rm L}t} + \varepsilon^{*}\hat{S}^{-}{\rm e}^{-{\rm i}\omega_{\rm L}t}, \tag{2}$$

where $\varepsilon = |\varepsilon|{\rm e}^{-{\rm i}\pi/2}$ stands for the complex amplitude of the classical field. As a result, the atom initially prepared in the excited state $|{\rm e}\rangle$ interacts with a classical field inside the cavity; after a time τ it evolves to the state

$$|\psi_{\rm A}(\tau)\rangle = \cos(|\varepsilon|\tau)|{\rm g}\rangle + \sin(|\varepsilon|\tau)|{\rm e}\rangle, \tag{3}$$

and setting $\tau = \pi/4|\varepsilon|$ one obtains

$$|\psi_{\rm A}\rangle = \frac{1}{\sqrt{2}}(|{\rm g}\rangle + |{\rm e}\rangle). \tag{4}$$

Second step: next tune the atomic transition frequency to ω_0 (hence far off resonance with the classical field), which is dispersive with respect to the cavity mode $\omega_{\rm c}$, detuned by $\Delta = \omega_0 - \omega_{\rm c}$. This is implemented via the Stark effect produced by a static electric field applied between the cavity mirrors [14]. The effective (dispersive) Hamiltonian for the atom–field system is [15]

$$\hat{H}_{\rm disp} = \frac{2g^2}{\Delta}\hat{a}^{\dagger}\hat{a}\hat{S}_z. \tag{5}$$

Suppose now the cavity field is initially prepared in a coherent state $|\alpha\rangle$ ($\alpha = r{\rm e}^{{\rm i}\phi}$), which is accomplished by connecting a microwave source to the cavity [16]. Thus, after an interaction time τ_1 the state describing the whole atom–field system evolves into

$$|\psi_{\rm AF}^{(1)}\rangle = |{\rm g}\rangle|{\rm e}^{{\rm i}\theta_1}\alpha\rangle + |{\rm e}\rangle|{\rm e}^{-{\rm i}\theta_1}\alpha\rangle, \tag{6}$$

with $\theta_1 = (2g^2/\Delta)\tau_1$. We note that in equation (6) the atom crossing the cavity produces a conditional negative (positive) phase shift in the field state when it is in the state $|{\rm e}\rangle$ ($|{\rm g}\rangle$).

Third step: remove the Stark effect on the atom and tune it again to the classical field during the same time τ as for the first step. So, after the interaction the atom–field state reads

$$|\psi_{\rm AF}^{(1)}\rangle' = |{\rm g}\rangle(|{\rm e}^{{\rm i}\theta_1}\alpha\rangle + |{\rm e}^{-{\rm i}\theta_1}\alpha\rangle) - |{\rm e}\rangle(|{\rm e}^{{\rm i}\theta_1}\alpha\rangle - |{\rm e}^{-{\rm i}\theta_1}\alpha\rangle). \tag{7}$$

Fourth step: detect the outgoing atom in the state $|{\rm g}\rangle$. This completes the set of four steps and projects the cavity field in the superposed state (up to normalization):

$$|\Psi_{\rm F}^{(1)}\rangle = |{\rm e}^{-{\rm i}\theta_1}\alpha\rangle + |{\rm e}^{{\rm i}\theta_1}\alpha\rangle. \tag{8}$$

Next, repeating the *four-step* procedure for a second atom, take again its initial state in $|{\rm e}\rangle$ and the initial field state now given by equation (8). So the final field state of the cavity, constructed in a *four-step* procedure, works as the initial state for the next *four-step* procedure. A straightforward calculation following the previous procedure, after detecting the second atom again in the state $|{\rm g}\rangle$, leads to the cavity field state (up to normalization):

$$|\psi_{\rm F}^{(2)}\rangle = |{\rm e}^{{\rm i}(\theta_1+\theta_2)}\alpha\rangle + |{\rm e}^{{\rm i}(\theta_1-\theta_2)}\alpha\rangle + |{\rm e}^{-{\rm i}(\theta_1-\theta_2)}\alpha\rangle + |{\rm e}^{{\rm i}(\theta_1+\theta_2)}\alpha\rangle \tag{9}$$

where $\theta_2 = (2g^2/\Delta)\tau_2$.

Proceeding further in this way, after the Nth repetition of the *four-step* procedure, plus all atomic detections in $|{\rm g}\rangle$, the field state generated in the cavity results:

$$|\psi_{\rm F}^{(N)}\rangle = \sum_{j=1}^{2^{N-1}}\left(|{\rm e}^{{\rm i}\gamma_j}\alpha\rangle + |{\rm e}^{-{\rm i}\gamma_j}\alpha\rangle\right), \tag{10}$$

with

$$\gamma_j = \sum_{k=1}^{N}\theta_{\rm k}(-1)^{{\rm B}_k^{(N)}(j-1)}, \tag{11}$$

where $B_k^{(N)}(j-1)$ is a function of $j-1$ and represents the kth position of the number $j-1$ in the binary form with N digits. For example, for $N = 3$ and $j = 3$, we have $B_1^{(3)}(2) = 0$, $B_2^{(3)}(2) = 1$, $B_3^{(3)}(2) = 0$, since the number 2 ($=j-1$) is represented by 010 in the binary form with three digits. Hence, according to equation (11) we obtain $\gamma_3 = \theta_1 - \theta_2 + \theta_3 - \ldots$.

Now, setting $\theta_k = \pi/2^k$ and $\phi = \pi/2^N$ one obtains

$$|\Psi_{\rm F}^{(N)}\rangle = \mathcal{N}_N(r)\sum_{j=0}^{J_N}(|-r{\rm e}^{\frac{{\rm i}\pi j}{2^{N-1}}}\rangle + |r{\rm e}^{\frac{{\rm i}\pi j}{2^{N-1}}}\rangle), \tag{12}$$

where $J_N = 2^{N-1} - 1$; $\mathcal{N}_N(r)$ stands for normalization, with $r = |\alpha|$. Equation (12) recovers the result obtained in equation (5) of [9].

The statistical distribution corresponding to the state given in equation (12) reduces to

$$P_N(n; r_{\rm max}) = \frac{2^N(r_{\rm max}^2)^n}{\mathcal{A}_N(r_{\rm max}^2)n!}\delta_{n,2^N}, \tag{13}$$

where

$$\mathcal{A}_N(r_{\rm max}^2) = \left[2^{2N}{\rm e}^{-r_{\rm max}^2}\mathcal{N}_N(r_{\rm max})^2\right]^{-1} \tag{14}$$

and $r_{\rm max}$ is the value of r yielding the Fock state $|2^N\rangle$ with maximum fidelity. Table 1 shows the optimal values of r producing the Fock states $|2^N\rangle$. Here note that the fidelity of our desired state is obtained from the statistical distribution $P_N(n; r_{\rm max})$, for $n = 2^N$.

Table 1. Optimal values of r and the associated fidelities of $|\Psi_{\rm F}^{(N)}\rangle$ and $|2^N\rangle$.

N	$\lvert 2^N\rangle$	$r_{\rm max}$	$P_N(2^N; r_{\rm max})$
2	$\lvert 4\rangle$	2.01	0.799
3	$\lvert 8\rangle$	2.61	0.982
4	$\lvert 16\rangle$	3.58	1.000
5	$\lvert 32\rangle$	4.96	1.000
6	$\lvert 64\rangle$	7.33	1.000

To be more specific we discuss the generation of the Fock state $|8\rangle$ using a Rydberg atom with the states $|{\rm g}\rangle$ and $|{\rm e}\rangle$ having principal quantum numbers 50 and 51 with a transition frequency 51.1 GHz [17]. We take typical values for the involved parameters in [17] for the coupling constant

Table 2. Values of the interaction times for the each atom to generate the Fock state $|8\rangle$.

kth atom	$\tau = \frac{\pi}{4\lvert\varepsilon\rvert}$ (μs)	θ_k	$\tau_k = \frac{\Delta}{2g^2}\theta_k$ (μs)	$\tau_{all} = 2\tau + \tau_k$ (μs)
1st	4	$\pi/2$	56	64
2rd	4	$\pi/4$	28	36
3th	4	$\pi/8$	14	22

$g \simeq 4.2\times10^5\ \mathrm{s}^{-1}$, the cavity length $L_c \simeq 1$ cm, $|\varepsilon| \simeq 2.5\times10^5$, damping time $t_{cav} \simeq 10^{-2}$ s [17, 18]. Setting the detuning $\Delta = 30g$, these data lead to the interaction times shown in table 2 for the successive atoms. This result requires Rydberg atoms with velocity $v_a \simeq 150$ m s^{-1} (which belongs to the typical interval available in laboratories: $v_a \sim$ 70–500 m s^{-1} [17]) since this velocity corresponds to the flight time inside the cavity of $L_c/v_a \simeq 67\ \mu$s, greater than the total interaction time for each atom. In table 1 we obtain $r = 2.61$ for the number state $|8\rangle$, which produces the decoherence time $t_d = t_{cav}/2r^2 \simeq 734\ \mu$s, greater than 122 μs (which is the sum of total interaction times for three atoms). So the scheme is experimentally feasible within the realm of microwaves.

In summary, we have discussed a scheme alternative to that of [10] creating a certain family of Fock states, $|2^N\rangle$. We are only concerned with the stationary case in [10]. The present method constitutes a convenient adaptation of a previous work introduced in [11], for the generation of arbitrary superposed coherent states over a circle in the phase space. A comparison of the present scheme with that in [10] is pertinent. First, they have in common the second and fourth steps, but the two Ramsey zones used in [10] are replaced here by the two interactions of a classical field with the atom crossing the cavity (first and third steps). Second, in both schemes one must control the atomic velocities; since they are different for different atoms in [10], and they are the same here, this seems to make the present scheme advantageous. However, this advantage may be only apparent, since the control of different velocities in [10] is replaced here by the control of the interaction times of the atom with the classical and quantized fields inside the cavity. Hence the present scheme is better than that in [10] whenever the control of successive atom–field interactions inside the cavity, proposed in [11], is easier than controlling the successive atomic velocities, used in [10]. Another pertinent comparison is one concerned with a previous method given by Szabo *et al* [8], which requires N atoms crossing the cavity to generate an arbitrary superposition of $N + 1$ coherent states and so would require $2^N - 1$ atoms to create the state $|\Psi_F^{(N)}\rangle$. Here, by dropping the generality, we obtain a specific superposition of 2^N coherent states using solely N atoms which, by appropriate choice of parameters, corresponds to the highly excited Fock state $|2^N\rangle$. Similar conclusions occur when comparing with [17]. Comparisons with a scheme given by Law and Eberly [19] and another given by Domokos *et al* [20] are also pertinent. In the first case, although only a single (three-level) atom is used to create a Fock state $|M\rangle$, the number of operations resulting is $2M$. For example, to create the state $|8\rangle$ one might realize 16 successive operations with the atom in the scheme [19], whereas in our scheme nine operations are realized. So, the scheme in [19] is more economic than ours as regards the number of atoms, but it is less economic than ours as regards the number of operations realized with the atom inside the cavity. In the second case [20], the authors also propose an interesting scheme for creating Fock states. They use a single atom which interacts with two modes of the same cavity by transferring photons from one of them to the other. The procedure has good accurary for Fock states around $|5\rangle$ whereas the present scheme is able to generate Fock states having higher excitations, e.g., $|8\rangle$, $|16\rangle$, $|32\rangle$, . . ., but at the expense of using a certain number of atoms, 3, 4, 5, . . ., respectively.

Acknowledgments

Partial support from the CNPq and FUNAPE-UFG, Brazilian agencies, is gratefully acknowledged.

References

[1] Zbinden H *et al* 2000 *J. Cryptol.* **13** 207
Gheri K M *et al* 1998 *Phys. Rev.* A **58** R2627
van Enk S J, Cirac J I and Zoller P 1997 *Phys. Rev. Lett.* **79** 5178
van Enk S J, Cirac J I and Zoller P 1998 *Science* **279** 205
[2] Jennewein T *et al* 2000 *Phys. Rev. Lett.* **84** 4729
[3] Caves C M and Drummond P D 1994 *Rev. Mod. Phys.* **66** 481
[4] Holland M J and Burnett K 1993 *Phys. Rev. Lett.* **71** 1355
[5] Varcoe B T H *et al* 2000 *Nature* **403** 743
Brattke S *et al* 2001 *Phys. Rev. Lett.* **86** 3534
[6] Bertet P *et al* 2002 *Phys. Rev. Lett.* **88** 143601
[7] Domokos P, Janszky J and Adam P 1994 *Phys. Rev.* A **50** 3340
Janszky J *et al* 1995 *Phys. Rev.* A **51** 4191
[8] Szabo S *et al* 1996 *Phys. Rev.* A **53** 2698
[9] Malbouisson J M C and Baseia B 1999 *J. Mod. Opt.* **46** 2015
[10] Maia L P A *et al* 2004 *J. Opt. B: Quantum Semiclass. Opt.* **6** 351
[11] Zheng S B and Guo G C 1998 *Phys. Lett.* A **244** 512
[12] Raimond J M, Brune M and Haroche S 2001 *Phys. Rev. Lett.* **73** 565
[13] Alsing P, Guo D S and Carmichael H J 1992 *Phys. Rev.* A **45** 5135
Dutra S M, Knight P L and Moya-Cessa H 1994 *Phys. Rev.* A **49** 1993
[14] Domokos P *et al* 1995 *Phys. Rev.* A **52** 3554
[15] Holland M J, Walls D F and Zoller P 1991 *Phys. Rev. Lett.* **67** 1716
[16] Raimond J M, Brune M and Haroche S 2001 *Rev. Mod. Phys.* **73** 565
[17] Brune M *et al* 1992 *Phys. Rev.* A **45** 5193
[18] Davidovich L *et al* 1993 *Phys. Rev. Lett.* **71** 2360
Davidovich L *et al* 1996 *Phys. Rev. Lett.* **53** 1295
[19] Law C K and Eberly H J L 1996 *Phys. Rev. Lett.* **76** 1055
[20] Domokos P *et al* 1998 *Eur. Phys. J.* D **1** 1