



# SU(2) Higher-order effective quark interactions from polarization



Fábio L. Braghin

Instituto de Física, Federal University of Goiás, Av. Esperança, s/n, 74.690-900, Goiânia, GO, Brazil

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## ABSTRACT

Higher order quark effective interactions are found for SU(2) flavor by departing from a non-local quark–quark interaction. By integrating out a component of the quark field, the determinant is expanded in chirally symmetric and symmetry breaking effective interactions up to the fifth order in the quark bilinears. The resulting coupling constants are resolved in the leading order of the longwavelength limit and exact numerical ratios between several of these coupling constants are obtained in the large quark mass limit. In this level, chiral invariant interactions only show up in even powers of the quark bilinears, i.e.  $\mathcal{O}(\bar{\psi}\psi)^{2n}$  ( $n = 1, 2, 3, \dots$ ), whereas (explicit) chiral symmetry breaking terms emerge as  $\mathcal{O}(\bar{\psi}\psi)^n$  being always proportional to some power of the Lagrangian quark mass.

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## 1. Introduction

The understanding of the effects and mechanisms by which quarks interact among themselves is a necessary step to provide a complete description of hadron structure and dynamics and the phase diagram of Strong Interactions. In low and intermediary energies these interactions can be parametrized in terms of realistic effective quark interactions that usually provide important information to establish the needed relations between QCD and hadron dynamics [1,2]. The basic and fundamental mechanisms that give rise to each of the effective interactions and parameters present in effective models and theories should be expected to be well understood, although a quite large amount of different quark effective interactions are expected to emerge due to the intricate structure of QCD. The Nambu Jona Lasinio (NJL) model is known to describe qualitatively well several important effects in hadron phenomenology [3,4] in spite of its known limitations. A large variety of possible corrections to the NJL coupling can be expected to emerge from QCD, and higher order quark interactions were shown to provide relevant effects for the ground state [5–7], chiral phase transition (flavor SU(2) and SU(3)) and higher energies [8–12] and eventually they might contribute to multi-quark structures [13]. In FAIR-GSI the high density phase diagram will be tested eventually providing relevant information also about the role of multi-quark interactions in different regions of the phase diagram. Few mechanisms have been shown to drive quark effective interactions by gluon exchange

[14–24]. Instanton mediation have been shown to provide one of the most investigated mechanisms for effective quark interactions for example by means of the Kobayashi–Maskawa–t Hooft interaction or instanton gas model. It depends strongly on flavor and, for flavor SU(2), it yields a second order quark interaction different from the usual chiral NJL interaction, producing the axial anomaly and its phenomenological consequences [14,25,3,4]. Polarization effects were shown to produce low energy and higher order effective interactions [22].

In the present work, flavor SU(2) higher order quark effective interactions are calculated from polarization effects by departing from a dressed one gluon exchange (i.e. a global color model) along the lines of Refs. [22]. Simple gluon exchange is a basic mechanism that cannot describe low energy hadron properties, including dynamical breakdown of chiral symmetry ( $D\chi$ SB), although it can be dressed by gluon interactions producing enough strength for  $D\chi$ SB [26–28]. This work is organized as follows. In the next section the method is shortly described according to which the quark bilinears are separated into two components, i.e.  $\bar{\psi}\Gamma\psi \rightarrow (\bar{\psi}\Gamma\psi)_1 + (\bar{\psi}\Gamma\psi)_2$ , as done in the background field method [29]. The background field ( $\psi_1$ ) remains as interacting quarks and the field  $\psi_2$  is integrated out. Instead of introducing auxiliary fields (a.f.) for the component that is integrated out, a weak field approximation is considered such that:  $(\bar{\psi}\psi)_1^2 \gg (\bar{\psi}\psi)_2^2$ . Results are the same as by introducing a.f. in the leading order since the a.f., for example as shown in Ref. [22,30,31], play no role in the resulting leading quark–quark effective interactions. The quark determinant is expanded in powers of quark bilinears yielding chiral invariant and also symmetry breaking terms proportional to the Lagrangian

E-mail address: [braghin@ufg.br](mailto:braghin@ufg.br).

quark mass. The corresponding effective couplings are resolved. This expansion is performed up to the eighth order for all the bilinears and up to the tenth order for the scalar-pseudoscalar ones. Some ratios between the effective coupling constant are shown to provide simple numerical values. Some numerical estimations are also shown.

## 2. Diquark interaction and quark field splitting

The departing point is the following quark effective interaction:

$$S_{eff}[\bar{\psi}, \psi] = \int_x \left[ \bar{\psi} (i\partial - m) \psi - \frac{g^2}{2} \int_y j_\mu^b(x) \tilde{R}_{bc}^{\mu\nu}(x-y) j_\nu^c(y) \right], \quad (1)$$

where  $b, c$  stand for color indices, the color quark current is  $j_b^\mu = \bar{\psi} \lambda_b \gamma^\mu \psi$ , the sum in color, flavor and Dirac indices are implicit,  $\int_x$  stands for  $\int d^4x$ , the kernel  $\tilde{R}_{bc}^{\mu\nu}$  can be written in terms of transversal and longitudinal components ( $R_T$  and  $R_L$ ) as:  $\tilde{R}_{ab}^{\mu\nu} \equiv \tilde{R}_{ab}^{\mu\nu}(x-y) = \delta_{ab} \left[ R_T \left( g^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{\partial^2} \right) + R_L \frac{\partial^\mu \partial^\nu}{\partial^2} \right]$  with implicit Dirac delta functions  $\delta(x-y)$ . With a Fierz transformation [3,4,30,31], by picking up the color singlet sector only, the above effective quark interaction can be expressed in terms of bilocal quark bilinears,  $j_i^q(x, y) = \bar{\psi}(x) \Gamma^q \psi(y)$  where  $q = s, p, v, a$  and  $\Gamma_q$  stands for Dirac and flavor SU(2) operators  $\Gamma_s = I$  for the  $2 \times 2$  flavor and  $4 \times 4$  identities,  $\Gamma_p = \sigma_i i \gamma_5$ ,  $\Gamma_v^\mu = \gamma^\mu \sigma_i$  and  $\Gamma_a^\mu = i \gamma_5 \gamma^\mu \sigma_i$ , being  $\sigma_i$  are the flavor SU(2) Pauli matrices. The Fierz transformed interaction is written as:  $\Omega = \alpha \sum_q j_i^q(x, y) R_q(x-y) j_i^q(y, x)$ , where  $\alpha = 8/9$ ,  $R_q$  are the kernels in each of the  $q$  channel of the interaction. Next the quark field is separated into two components, one of them associated with polarization virtual processes eventually to the formation of quark bound states such as light mesons and the chiral condensate and the other component remains as (constituent) quark. This procedure is basically the one loop background field method [29], and this will be done by rewriting the quark bilinears above as:

$$\bar{\psi} \Gamma^q \psi \rightarrow (\bar{\psi} \Gamma^q \psi)_2 + (\bar{\psi} \Gamma^q \psi)_1. \quad (2)$$

The Fierz transformed non-local interaction above can then be written as:  $\Omega \rightarrow \Omega_1 + \Omega_2 + \Omega_{12}$  where  $\Omega_1$  and  $\Omega_2$  stand for the interactions of each of the quark components, and  $\Omega_{12}$  for the mixed terms. The component  $\psi_2$  will be integrated out and the fourth order terms can be eliminated in different approximated ways. Firstly by simply considering a weak field approximation and therefore by neglecting  $\Omega_2 \ll \Omega_1$ . This yields the same results as the leading terms resulting from the auxiliary field method which eliminates the fourth order interactions  $\Omega_2$ , as discussed in Refs. [22,30,31]. In this case, bilocal auxiliary fields ( $S, P_i, V_\mu^i, \bar{A}_\mu^i$ ) are introduced which couple to the remaining quark component. These couplings encode the non-linearities of the initial model. However in this work we are interested only in the quark self-interactions and these couplings can be neglected. Even if one were interested in the effective interactions induced by these couplings to the auxiliary fields (a.f.), the resulting quark–quark effective interactions induced by the a.f. would be of higher order and numerically smaller. By integrating out the component  $(\psi)_2$ , and by writing the determinant as:  $\det(A) = \exp(\text{Tr} \ln A)$ , the following non-linear non-local effective action for quarks  $(\psi)_1$  is obtained:

$$S_{eff} = -i \text{Tr} \ln \left\{ i(S_0)^{-1}(x-y) - i\alpha g^2 \bar{R}^{\mu\nu}(x-y) \gamma_\mu \sigma_i \left[ (\bar{\psi}_y \gamma_\nu \sigma_i \psi_x) - i\gamma_5 (\bar{\psi}_y i \gamma_5 \gamma_\nu \sigma_i \psi_x) \right] + 2i\alpha g^2 R(x-y) \left[ (\bar{\psi}_y \psi_x) + i\gamma_5 \sigma_i (\bar{\psi}_y i \gamma_5 \sigma_i \psi_x) \right] \right\} - I_0, \quad (3)$$

where  $\text{Tr}$  stands for traces of discrete internal quantum numbers indices and integration of spacetime coordinates/momentum and  $I_0 = \int_x \left[ \bar{\psi} (i\gamma \cdot \partial - m) \psi - \frac{g^2}{2} \int_y j_\mu^a(x) R_{ab}^{\mu\nu}(x-y) j_\nu^b(y) \right]$ . In this expression the label 1 for the quark field was omitted because it is the only one remaining from here on.  $(S_0)^{-1} = (S_0)^{-1}(x-y) \equiv (i\gamma \cdot \partial - m)$ , with an implicit Dirac delta function, and where instead of  $m$  one could introduce an effective mass ( $m^*$ ) which arise from the coupling to the scalar auxiliary variable  $s$  which produces the dynamical chiral symmetry breaking as discussed at length in Refs. [3,4,22,30,31]. The following kernels have also been defined from the Fierz transformation:  $R = R(x-y) = 3R_T + R_L$  and  $\bar{R}^{\mu\nu} = \bar{R}^{\mu\nu}(x-y) = g^{\mu\nu}(R_T + R_L) + 2 \frac{\partial^\mu \partial^\nu}{\partial^2} (R_T - R_L)$  with implicit Dirac delta functions. By neglecting the derivative couplings, with a shorthand notation for which the non-local character of all the kernels is omitted, i.e.  $R = R(x-y)$ ,  $\bar{R}^{\mu\nu} = \bar{R}^{\mu\nu}(x-y)$  and  $S_0 = S_0(x-y)$ , the quark determinant above can be rewritten [32] as:

$$I_d \equiv -\frac{i}{2} \text{Tr} \ln \left[ S^{-1} S^\dagger^{-1} \right] = -\frac{i}{2} \text{Tr} \ln \left[ \tilde{S}_0^{-1} \right] - \frac{i}{2} \text{Tr} \ln \left[ 1 + \beta \tilde{S}_0 \left( 2R \bar{\psi} \psi - \bar{R}^{\mu\nu} \gamma_\mu \sigma_i \bar{\psi} \gamma_\nu \sigma_i \psi \right) + g^4 \sum_{q,q'} \tilde{S}_0 a_{q,q'} (\Gamma_q \bar{\psi} \Gamma_q \psi) (\Gamma_{q'}^\dagger \bar{\psi} \Gamma_{q'} \psi) \right], \quad (4)$$

where  $\beta = 2mg^2\alpha$  was defined for the quark mass (symmetry breaking term),  $\tilde{S}_0 \equiv \tilde{S}_0(x-y) = -1/(\partial^2 + m^2)\delta(x-y)$  was factorized producing an irrelevant multiplicative constant in the generating functional,  $a_{q,q'}$  are coefficients for each of the flavor channels, and crossed terms ( $q, q' = s, p, v, a$ ) with the corresponding operators  $\Gamma_q$  and kernels  $R_q$ . This expression still has a strong non-local character which is not written explicitly. This determinant will be expanded for small  $\tilde{S}_0$ , i.e. large quark (effective) mass by considering that  $m$  may be an effective (constituent) quark mass. A small coupling  $g^2$  or weak quark field  $\psi_1$  yields essentially the same results such that the final polynomial quark effective interactions are written in terms of effective coupling constants in the local limit of the resulting couplings. It can be noticed that all the chiral invariant interactions only appear from the contributions exclusively of the last term inside of the determinant. Therefore chiral invariant terms for this SU(2) flavor will be  $\mathcal{O}[(\bar{\psi}\psi)^2]^n$ . All the interactions for which the second term contributes (proportional to the quark mass) will be not chiral invariant. One of the first order terms yields a contribution for the quark effective mass [22] of the form:  $\Delta m^* = -i2\alpha g^2 m \text{Tr} \tilde{S}_0 R$ .

## 3. SU(2) Quark effective interactions

The leading terms, by resolving the effective coupling constants in the longwavelength limit and the zero order derivative expansion, are:

$$\mathcal{L}_4 = g_4 \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} \sigma_i i \gamma_5 \psi)^2 \right] - g_{v4} \left[ (\bar{\psi} \sigma_i \gamma_\mu \psi)^2 + (\bar{\psi} \sigma_i \gamma_5 \gamma_\mu \psi)^2 \right] + \mathcal{L}_4^{sb} \quad (5)$$

where  $\mathcal{L}_4^{sb} = g_{4, sb} (\bar{\psi} \psi)^2 + g_{4, v, sb} (\bar{\psi} \sigma_i \gamma_\mu \psi)^2$  are symmetry breaking terms which emerge from the second order expansion although they are of the same order of magnitude as the first one, as it can be noted in the next expressions. These effective coupling constants were resolved as:

$$g_4(1; \delta_{ij}) = -i2(g^2\alpha)^2 N_c Tr'' \tilde{S}_0 R^2(1; \sigma_i \sigma_j), \quad (6)$$

$$g_{4, sb} = i4(g^2\alpha)^2 N_c Tr'' m^2(\tilde{S}_0 R)^2, \quad (7)$$

$$g_{v4} \delta_{ij} g^{\mu\nu} = -\frac{i}{2}(g^2\alpha)^2 N_c Tr'' \tilde{S}_0 \bar{R}^{\mu\rho} \bar{R}^{\nu\sigma} (\sigma_i \sigma_j) \gamma_\rho \gamma_\sigma \quad (8)$$

$$g_{4, v, sb} \delta_{ij} g^{\mu\nu} = i(g^2\alpha)^2 N_c Tr'' m^2 \tilde{S}_0 \bar{R}^{\mu\rho} \tilde{S}_0 \bar{R}^{\nu\sigma} (\sigma_i \sigma_j) \gamma_\rho \gamma_\sigma, \quad (9)$$

where  $Tr''$  includes all the traces in internal and spacetime indices except the trace in color indices that has already been done. The couplings with  $g_4$  and  $g_{v4}$  are the usual NJL and vector NJL couplings respectively with dimension  $1/M^2$  for a mass scale  $M$ . For the class of diagrams of this one fermion loop level, by considering that  $g^2 \sim \tilde{g}^2/N_c$ , the resulting  $n$ -quark coupling constants are of the order of  $N_c^{1-n}$  in agreement with [33].

The non-derivative sixth order terms, after resolving the effective coupling constants, are all symmetry breaking and they were found to be:

$$\begin{aligned} \mathcal{L}^{(6)} = & g_{6, sb}^{(1)} (\bar{\psi} \psi) \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} \sigma_i i \gamma_5 \psi)^2 \right] \\ & - g_{6, sb, a} \epsilon_{ijk} (\bar{\psi} \sigma_i i \gamma_5 \gamma_\mu \psi) (\bar{\psi} \sigma_j \gamma^\mu \psi) (\bar{\psi} \sigma_k i \gamma_5 \psi) \\ & - g_{6, sb, a} \left[ (\bar{\psi} \sigma_i \gamma_\mu \psi)^2 + (\bar{\psi} \sigma_i i \gamma_5 \gamma_\mu \psi)^2 \right] (\bar{\psi} \psi) \\ & + g_{6, sb}^{(3)} (\bar{\psi} \psi)^3 \end{aligned} \quad (10)$$

where

$$g_{6, sb}^{(1)}(1; \delta_{ij}) = i2(\alpha g^2)^3 N_c Tr'' m \tilde{S}_0 R (\tilde{S}_0 R^2)(1; \gamma_5^2 \sigma_i \sigma_j),$$

$$g_{6, sb}^{(3)}(1; \delta_{ij}) = -i \frac{32}{3} (\alpha g^2)^3 N_c Tr'' m^3 (\tilde{S}_0 R)^3(1; \sigma_i \sigma_j),$$

$$\begin{aligned} g_{6, sb, a} g^{\nu\sigma}(\delta_{ij}; i \epsilon_{ijk}) \\ = i(\alpha g^2)^3 N_c Tr'' m \tilde{S}_0 R \tilde{S}_0 R^{\mu\nu} R^{\rho\sigma} \gamma_\mu \gamma_\rho \gamma_5^2 \sigma_i \sigma_j (1; \sigma_k), \end{aligned} \quad (11)$$

where for further calculation one defines  $\bar{R}_2^{\nu\sigma} = \bar{R}^{\mu\nu} \bar{R}_\mu^\sigma = (R_T + R_L)^2 g^{\nu\sigma} + 8R_T(R_T - R_L) \frac{\partial^{\nu\sigma}}{\partial^2}$ .

There are several chiral invariant and symmetry breaking non-derivative eighth order interactions. They were found to be:

$$\begin{aligned} \mathcal{L}^{(8)} = & g_8 \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} i \gamma_5 \sigma_i \psi)^2 \right]^2 \\ & + g_{8, sb}^{(2)} (\bar{\psi} \psi)^2 \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} \sigma_i i \gamma_5 \psi)^2 \right] + g_{8, sb}^{(4)} (\bar{\psi} \psi)^4 \\ & + g_{8v} \left[ (\bar{\psi} \gamma_\mu \sigma_i \psi)^2 + (\bar{\psi} \gamma_5 \gamma_\mu \sigma_i \psi)^2 \right]^2 \\ & - g_{8v, sb} (\bar{\psi} \psi)^2 \left[ (\bar{\psi} \gamma_\mu \sigma_i \psi)^2 + (\bar{\psi} \gamma_5 \gamma_\mu \sigma_i \psi)^2 \right] \\ & - g_{8vs} \left[ (\bar{\psi} \gamma_\mu \sigma_i \psi)^2 + (\bar{\psi} \gamma_5 \gamma_\mu \sigma_i \psi)^2 \right] \left[ (\bar{\psi} \psi)^2 \right. \\ & \left. + (\bar{\psi} i \gamma_5 \sigma_i \psi)^2 \right] \\ & + g_8^s (\bar{\psi} \gamma_\nu \sigma_j \psi)^2 \left[ (\bar{\psi} \gamma_\mu \sigma_i \psi)^2 + (\bar{\psi} \gamma_5 \gamma_\mu \sigma_i \psi)^2 \right], \end{aligned} \quad (12)$$

where the chiral invariant terms are of second order of the expansion, and the symmetry breaking are of third and fourth orders in the expansion of  $I_d$ . Up to this order of the expansion, terms in odd powers of the pseudoscalar and axial bilinears naturally disappear due to the traces such as  $tr(\gamma_5) = 0$ . The effective coupling constants are the following:

$$g_8(1; \delta_{ij}) = 4i(\alpha g^2)^4 N_c Tr'' (\tilde{S}_0 R^2)^2(1; \gamma_5^2 \sigma_i \sigma_j),$$

$$g_{8, sb}^{(2)}(1; \delta_{ij}) = 128i(\alpha g^2)^4 N_c Tr'' m^2 (\tilde{S}_0 R)^2 (\tilde{S}_0 R^2)(1; \gamma_5^2 \sigma_i \sigma_j),$$

$$g_{8, sb}^{(4)} = 64i(\alpha g^2)^4 N_c Tr'' m^4 (\tilde{S}_0 R)^4,$$

$$\begin{aligned} g_{8v} \Gamma^{\mu_1 \nu_1 \rho_1 \sigma_1} \Gamma_{ijkl} = & \frac{i}{2} (\alpha g^2)^4 N_c Tr'' \tilde{S}_0 \bar{R}^{\mu_1 \mu_2} \bar{R}^{\nu_1 \nu_2} \tilde{S}_0 \bar{R}^{\rho_1 \rho_2} \bar{R}^{\sigma_1 \sigma_2} \\ & \times (\gamma_{\mu_2} \gamma_{\nu_2} \gamma_{\rho_2} \gamma_{\sigma_2}) (\sigma_i \sigma_j \sigma_k \sigma_l), \end{aligned}$$

$$g_{8vs} g^{\mu\rho} \delta_{ij} = -i4(\alpha g^2)^4 N_c Tr'' (\tilde{S}_0 R^2) (\tilde{S}_0 R^{\mu\nu} R^{\rho\sigma}) \sigma_i \sigma_j \gamma_\nu \gamma_\sigma,$$

$$\begin{aligned} g_{8v, sb} g^{\mu\rho} \delta_{ij} = & -i8(\alpha g^2)^4 N_c Tr'' m^2 (\tilde{S}_0 R)^2 \\ & \times (\tilde{S}_0 R^{\mu\nu} R^{\rho\sigma}) \sigma_i \sigma_j \gamma_\nu \gamma_\sigma, \end{aligned}$$

$$\begin{aligned} g_8^{(s)} \Gamma_{\mu\rho}^{\mu_1 \rho_1} \delta_{ijkl} = & -\frac{i}{2} (\alpha g^2)^4 N_c Tr'' m^2 (\tilde{S}_0 R^{\mu_1 \nu_1}) (\tilde{S}_0 R^{\rho_1 \sigma_1}) \\ & \times (\tilde{S}_0 R_\mu^\nu R_\rho^\sigma) \gamma_{\nu_1} \gamma_{\sigma_1} \gamma_\nu \gamma_\sigma (\sigma_i \sigma_j \sigma_k \sigma_l), \end{aligned} \quad (13)$$

where  $\Gamma_{ijkl} = \delta_{ij} \delta_{kl} + \delta_{il} \delta_{jk} - \delta_{ik} \delta_{jl}$  and  $\Gamma_{\mu\nu\rho\sigma} = g_{\mu\nu} g_{\rho\sigma} + g_{\mu\sigma} g_{\nu\rho} + g_{\mu\rho} g_{\nu\sigma}$ . Some of these terms were considered in Ref. [9].

The tenth order interaction terms (leading terms from expansion up to the fifth order) are all symmetry breaking and the scalar-pseudoscalar terms can be written as:

$$\begin{aligned} \mathcal{L}^{(10)} = & g_{10}^{(1)} (\bar{\psi} \psi) \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} \sigma_i i \gamma_5 \psi)^2 \right]^2 \\ & + g_{10}^{(3)} (\bar{\psi} \psi)^3 \left[ (\bar{\psi} \psi)^2 + (\bar{\psi} \sigma_i i \gamma_5 \psi)^2 \right] + g_{10}^{(5)} (\bar{\psi} \psi)^5, \end{aligned} \quad (14)$$

where:

$$g_{10}^{(1)} = -\frac{i}{2} (4\alpha g^2)^5 N_c Tr'' m (\tilde{S}_0 R) (\tilde{S}_0 R^2)^2,$$

$$g_{10}^{(3)} = \frac{i3}{4} (4\alpha g^2)^5 N_c Tr'' m^3 (\tilde{S}_0 R)^3 (\tilde{S}_0 R^2),$$

$$g_{10}^{(5)} = -\frac{i}{10} (4\alpha g^2)^5 N_c Tr'' m^5 (\tilde{S}_0 R)^5. \quad (15)$$

The symmetry breaking terms of the scalar-pseudoscalar channel can be written in a general form for the  $n$ -term of the expansion in terms of a number (combinatorial)  $a_m$ :

$$g_{2n, sb}^{(m)} = \frac{i}{n} a_m (2\alpha g^2)^n Tr m^m (\tilde{S}_0^{\frac{(n+m)}{2}} R^n). \quad (16)$$

One can consider two particular limits for calculating ratios of the quark effective coupling constants depending on the gluon propagator components. These ratios are obtained by assuming a large quark mass and by choosing one of the two following limits: (I)  $R_L = 0$  ( $T$ ), or (II)  $R_T = 0$  ( $L$ ). With the expressions shown above which turns out to depend on the vector or axial bilinears, the moduli of some ratios yield:

$$\begin{aligned} \left| \frac{g_4}{g_{v4}} \right|^T \sim \left| 4 \frac{g_4}{g_{4, v, sb}} \right|^T \sim 3, \quad \left| \frac{g_{6, sb}^{(1)}}{g_{6, sb, a}} \right|^T \sim 6, \\ \left| \frac{g_8}{g_{8vs}} \right|^T \sim \frac{3}{4}, \end{aligned} \quad (17)$$

$$\begin{aligned} \left| \frac{g_4}{g_{v4}} \right|^L \sim \left| 4 \frac{g_4}{g_{4, v, sb}} \right|^L \sim 1, \quad \left| \frac{g_{6, sb}^{(1)}}{g_{6, sb, a}} \right|^L \sim 2, \\ \left| \frac{g_8}{g_{8vs}} \right|^L \sim \frac{1}{4}. \end{aligned} \quad (18)$$

The ratios between the chiral invariant fourth order coupling constants ( $\frac{g_4}{g_{v4}}$ ) are in good agreement with phenomenology [34–36]. These ratios might therefore present quite strong gauge dependence and this issue will not be discussed in the present work. Some ratios are independent of the gluon kernel component and their moduli are given by:

$$\left| \frac{g_4}{g_{4, sb}} \right| \sim \frac{1}{2}, \quad \left| \frac{g_{6, sb}^{(1)}}{g_{6, sb}^{(3)}} \right| \sim \frac{3}{4}, \quad \left| \frac{g_8}{g_{8sb}^{(2)}} \right| \sim \left| \frac{g_8}{2g_{8sb}^{(4)}} \right| \sim \frac{1}{32},$$

$$\left| \frac{g_{10}^{(1)}}{g_{10}^{(3)}} \right| \sim 1, \quad \left| \frac{g_{10}^{(1)}}{g_{10}^{(5)}} \right| \sim 5, \quad (19)$$

the first of this ratios shows that the exclusive contribution of the explicit chiral symmetry breaking via the Lagrangian quark mass for the coupling  $(\bar{\psi}\psi)^2$  is of the same order of magnitude as the NJL coupling. Next, some numerical values are shown by replacing the traces in spacetime coordinates by momentum integration rotated to Euclidean space in the limit of zero momentum exchange. A simplified confining gluon propagator from Ref. [26] is considered with the same values for the prescription given by expression (10) of Ref. [26]. The only ultraviolet divergent effective parameter presented above is the one for the effective mass correction before Section 3. It can be directly renormalized with the Lagrangian mass counterterms and it will not be estimated here. The mass for the quark kernel  $\tilde{S}_0$  was considered to be an effective mass from  $D\chi SB$   $m = 0.33$  GeV and the coupling constant  $g^2$  as the zero momentum limit of the QCD lattice calculations divided by 1000, i.e.  $g^2 = 17.8\pi/(10^3 N_c)$  from Ref. [37]. It is reasonable to consider a reduced value because a full running coupling constant would reduce the contribution of the higher energy modes. The resulting values were found to be  $g_4 \simeq 1.2$  GeV<sup>-2</sup>,  $g_6 \simeq -28.2$  GeV<sup>-5</sup>,  $g_8 \simeq 4.1 \cdot 10^4$  GeV<sup>-8</sup> and  $g_{10}^{(1)} \simeq 2.2 \cdot 10^8$  GeV<sup>-11</sup>. These values are comparable to values obtained in the literature by phenomenological fitting except the higher order ones. From Ref. [9] some SU(2) flavor coupling constants were considered as:  $g_4 \sim 10$  GeV<sup>-2</sup> and  $g_8 \sim 100$ –450 GeV<sup>-8</sup>, and for the sake of comparison for SU(3) Refs. [6]  $g_4 \sim 10$  GeV<sup>-2</sup>,  $g_6 \sim -1100$  GeV<sup>-5</sup>,  $g_8 \sim 6000$  GeV<sup>-8</sup>. The values for the higher order couplings are somewhat larger than the values obtained from phenomenology and this might be related to the truncated momentum dependence considered and to the values of the parameters  $m$ ,  $g^2$  considered above.

The emerging quark–quark potential is therefore composed by several types of chiral invariant and symmetry breaking terms and this intricate structure is expected from a confining theory [2]. Obvious corrections to the effective interactions found above are due to the derivative interactions that were not calculated and which may be expected to be relevant for a complete effective theory for quark dynamics. It is interesting to emphasize two points: firstly it can be seen in expressions (7), (9), (11) and the symmetry breaking couplings of expressions (13) and (15), that all the symmetry breaking effective interactions have the effective couplings proportional to the Lagrangian quark mass, that is the explicit symmetry breaking term. If the quark mass were corrected by the quark condensate to an effective quark mass the same conclusion holds. Secondly, the strength of the resulting symmetry breaking effective couplings are of the order of the chiral invariant terms. The expressions for these effective quark interactions were obtained without an explicit form of the gluon propagator which plays a fundamental role in the resulting relative strength of the resulting effective coupling constants. Furthermore all the expressions for the effective coupling constants were written in a way to make possible to compute the corresponding form factors. It is also interesting to emphasize that results of this work allows for systematic computation of effective coupling constants without performing extensive phenomenological fits with hadron masses and/or couplings. Although the gluon propagator and higher order gluon interactions in the departing quark effective action might induce different

quark–quark effective interactions they should not be expected to change the shape of the effective interactions found in the present work.

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