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Research Article

Application of Three Bioinspired Optimization Methods for the Design of a Nonlinear Mechanical System

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The present work focuses on the optimal design of nonlinear mechanical systems by using heuristic optimization methods. In this context, the nonlinear optimization problem is devoted to a two-degree-of-freedom nonlinear damped system, constituted of a primary mass attached to the ground by a linear spring and a secondary mass attached to the primary system by a nonlinear spring. This arrangement forms a nonlinear dynamic vibration absorber (nDVA), which is used in this contribution as a representative example of a nonlinear mechanical system. The sensitivity analysis of the suppression bandwidth, namely, the frequency range over which the ratio of the main mass displacement amplitude to the amplitude of the forcing function is less than unity, with respect to the design variables that characterize the nonlinear system based on the first order finite differences is presented. For illustration purposes the optimization problem is written as to maximize the suppression bandwidth by using three recent bioinspired optimization methods: Bees Colony Algorithm, Firefly Colony Algorithm, and Fish Swarm Algorithm. The results are compared with other evolutionary strategies.

1. Introduction

In various mechanical design contexts, engineers have to deal with nonlinear systems in which the dynamic response depends on a number of physical parameters. An example of such a system is the so-called dynamic vibration absorber (DVA). The DVA is used to reduce noise and vibration in various types of engineering systems, such as compressors, robots, ships, power lines, airplanes, and helicopters. Much of the knowledge available to date is compiled in the original patent by Frahm [1], in the books by Hartog [2] and Koronev and Reznikov [3] and in some review works such as the one by Steffen Jr and Rade [4]. In the last two decades, a great deal of effort has been devoted to the development of mathematical models for characterizing the mechanical behavior of nonlinear dynamic vibration absorbers (nDVAs) accounting for their typical dependence on design parameters that influence the nonlinear behavior of the system. A particular type of nDVA is the so-called viscoelastic neutralizer as studied by Espíndola and Bavastri [5]. Borges et al. [6] proposed and determined the robust optimal design of an nDVA combining sensitivity analysis and multiobjective optimization. Different techniques have been proposed to design viscoelastic vibration absorbers [7, 8]. Besides the well-known complexity of the modeling strategy involved in nonlinear dynamics, some general methodologies have been suggested and have been shown to be particularly suitable to be used in combination with structural systems discretization. This aspect makes them very attractive for the modeling of nonlinear dynamic vibration absorbers. Among these strategies, the theoretical study proposed by Nissen and coworkers [9] and Pai and Schulz [10] should be mentioned, in which techniques to improve the stability and efficiency of nDVAs into a frequency band of interest have been proposed, leading to refined nDVAs. Also, Rice and McCraith [11] and Shaw and coworkers [12] suggested optimization strategies to be applied to the design of nDVAs by applying an asymmetric nonlinear Duffing-type element incorporated in the suspension for narrow-band absorption applications.

Nowadays, different approaches based on optimization methods have been proposed to solve mechanical design problems. Cao and Wu [13] proposed a cellular automata based genetic algorithm applied to mechanical systems design. In this algorithm, the individuals in the population are mapped onto a cellular automata to realise the locality and neighbourhood. The mapping is based on the individuals' fitness and the Hamming distances between individuals. The selection of individuals is controlled based on the structure of cellular automata, to avoid the fast population diversity loss and improve the convergence performance during the genetic search. He and coworkers He, Prempain, and Wu [14] proposed an improved particle swarm optimizer (PSO) associated with the fly-back mechanism to maintain a feasible population. This optimization strategy was applied to mechanical systems design involving problem-specific constraints and mixed variables such as integer, discrete, and continuous variables. In this sense, biological systems have contributed significantly to the development of new optimization techniques. These methodologies, known as Bioinspired Optimization Methods (BiOMs), are based on strategies that seek to mimic the behavior observed in species found in the nature to update a population of candidates to solve optimization problems [15, 16]. These systems have the capacity to notice and modify their environment in order to seek for diversity and convergence. In addition, this capacity makes possible the communication among the agents (individuals of population) that capture the changes in the environment generated by local interactions [17].

In this context, nature-inspired algorithms have contributed significantly to the development of new optimization techniques. Among the most recent bioinspired strategies stand the Bees Colony Algorithm (BCA) [18], the Firefly Colony Algorithm (FCA) [19], and the Fish Swarm Algorithm (FSA) [20]. The BCA is based on the behavior of bees colonies in their search of raw materials for honey production. According to Lucic and Teodorovic [21], in each hive groups of bees (called scouts) are recruited to explore new areas in search for pollen and nectar. These bees, returning to the hive, share the acquired information so that new bees are indicated to explore the best regions visited in an amount proportional to the previously passed assessment. Thus, the most promising regions are best explored and eventually the worst end up being discarded. Every iteration, this cycle repeats itself with new areas being visited by scouts. The FCA mimics the patterns of short and rhythmic flashes emitted by fireflies in order to attract other individuals to their vicinities. The corresponding optimization algorithm is formulated by assuming that all fireflies are unisex, so that one firefly will be attracted to all other fireflies. Attractiveness is proportional to their brightness, and for any two fireflies, the less bright will attract (and thus move to) the brighter one. However, the brightness can decrease as their distance increases and if there are no fireflies brighter than a given firefly it will move randomly. The brightness is associated with the objective function for optimization purposes [19]. Finally, the FSA is a random search algorithm based on the behaviour of fish swarm observed in nature. This behavior can be summarized as follows [20]: *random behavior*—in general, fish looks at random for food and other companion; *searching behavior*—when the fish discovers a region with more food, it will go directly and quickly to that region; *swarming behavior*—when swimming, fish will swarm naturally in order to avoid danger; *chasing behavior*—when a fish in the swarm discovers food, the others will find the food dangling after it; *leaping behavior*—when fish stagnates in a region, a leap is required to look for food in other regions.

In this way, the present work is dedicated to presenting alternative techniques for the optimal design of a nonlinear dynamic vibration absorber (nDVA) by using heuristic optimization methods to maximize the suppression bandwidth. For this aim, this contribution focuses on the theoretical study and numerical simulation of a two-degree-of-freedom nonlinear damped system, constituted of a primary mass attached to the ground by a linear spring and a secondary mass attached to the primary system by a nonlinear spring (nDVA). A previous contribution regarding this type of mechanical system can be found in Borges et al. [6]. Then, the three previously mentioned optimization techniques are applied to the design of an nDVA for illustration purposes; however, they are intended to be general in the sense that they can be applied to design different types of nonlinear mechanical systems. This work is organized as follows. The mathematical formulation of the nonlinear dynamic system and sensitivity analysis are presented in Sections 2 and 3, respectively. A review of the BCA, the FCA, and FSA are presented in Section 4. The results and discussion are described in Section 5. Finally, the conclusions and suggestions for future work conclude the paper.

2. Mathematical Modeling of the Nonlinear Dynamic System

Consider the two-degree-of-freedom (d.o.f.) model shown in Figure 1.

This device consists of a damped primary system attached to the ground by a suspension that includes either a linear or a nonlinear spring, and a damped secondary mass coupled to the primary system by a spring with nonlinear characteristics. The external force applied to the primary system is given by the following expression:

$$F_1 = p\cos(\omega t). \qquad (1)$$

The constitutive forces of the springs are given by

$$r_i(x_i) = K_i x_i + k_i^{\mathrm{nl}} x_i^3 \quad i = 1, 2, \qquad (2)$$

where x_1 represents the displacement of the primary system with respect to the ground, and x_2 is the displacement of

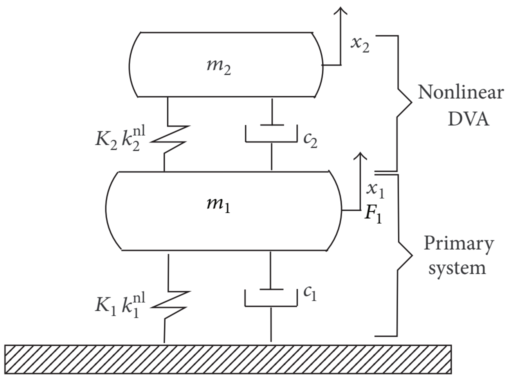

Figure 1: Two degree-of-freedom damped system [6].

the DVA with respect to the primary system. In the model above, the dampers are linear, however springs have nonlinear characteristics, where K_i and k_i^{nl} represent, respectively, the linear and nonlinear coefficients of the springs.

With the aim of obtaining the dimensionless normalized equations of motion for the nonlinear system, the displacements are normalized with respect to the length of a given vector x_c [22] according to the following expression:

$$y_i = \frac{x_i}{x_c}. \tag{3}$$

In addition, one introduces the following relations to the system:

$$\begin{gathered} \overline{\omega}^2 = \frac{k_i}{m_i}, \qquad \zeta_i = \frac{c_i}{2\sqrt{k_i m_i}}, \\ \delta_i = 2\zeta_i\omega_i, \qquad \eta_i = \omega_i^2, \qquad \varepsilon_i = \frac{k_i^{nl} x_c^2}{m_i \omega^2}, \\ \rho = \frac{\omega_2}{\omega_1}, \qquad P = \frac{p}{m_1 \overline{\omega}^2 x_c}, \\ \Omega = \frac{\omega}{\overline{\omega}_1}, \qquad \mu = \frac{m_2}{m_1}. \end{gathered} \tag{4}$$

By applying Newton's second law, and after some algebraic manipulations, the following normalized equation of motion of the nonlinear dynamic system can be expressed under the following matrix form:

$$\mathbf{M}\ddot{\mathbf{y}}(t) + \mathbf{C}\dot{\mathbf{y}}(t) + \mathbf{K}\mathbf{y}(t) = \mathbf{f}(t), \tag{5}$$

where the normalized mass, damping and stiffness matrices are expressed, respectively, by the following relations:

$$\begin{gathered} \mathbf{M} = \begin{bmatrix} 1+\mu & \mu \\ \mu & \mu \end{bmatrix}, \qquad \mathbf{C} = \begin{bmatrix} \delta_1 & 0 \\ 0 & \mu\delta_2 \end{bmatrix}, \\ \mathbf{K} = \begin{bmatrix} \eta_1 & 0 \\ 0 & \eta_2 \end{bmatrix}. \end{gathered} \tag{6}$$

The normalized displacement and force vectors are given by:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}, \qquad \mathbf{f} = \begin{bmatrix} P\cos(\tau)\,\varepsilon_1 y_1^3 \\ -\varepsilon_2 y_2^3 \mu \end{bmatrix}. \tag{7}$$

It is important to emphasized that the present contribution is dedicated to maximize the suppression bandwidth.

2.1. Steady-State Response of the Nonlinear System. In this paper, the Krylov-Bogoliubov method [23] is used to integrate the matrix equation of motion (1). This method leads to an approximate solution of nonlinear differential equations. The process is based on the following transformation of variables:

$$\mathbf{y} = \mathbf{u}(\tau)\cos(\tau) + \mathbf{v}(\tau)\sin(\tau), \tag{8}$$

where $\tau = \omega t$ is the time dependence of $\mathbf{u} = (u_1 u_2)^T$, and $\mathbf{v} = (v_1 v_2)^T$ is assumed to be small for high-order terms, such as the vectors $\mathbf{u}$ and $\mathbf{v}$.

After mathematical manipulation, a nonlinear algebraic system with four equations and four variables u_1, u_2, v_1 and v_2 is obtained as follows:

$$\begin{gathered} \left(1+\mu-\omega_1^2\right)u_1 + \mu u_2 - 2\zeta_1\omega_1 v_1 \\ - \frac{3\varepsilon_1\left(u_1^2+v_1^2\right)u_1}{4} + \beta\omega_1^2 = 0, \\ \mu u_1 + \left(\mu - \mu\rho^2\omega_1^2\right)u_2 - \mu\left(2\zeta_2\rho\omega_1^2 v_2 + \frac{3\varepsilon_2\left(u_2^2+v_2^2\right)u_2}{4}\right) \\ \left(\omega_1^2 - 1 - \mu\right)v_1 - \mu v_2 - 2\zeta_1\omega_1 u_1 + \frac{3\varepsilon_1\left(u_1^2+v_1^2\right)v_1}{4} = 0, \\ \mu v_1 + \left(\mu\rho^2\omega_1^2 - \mu\right)v_2 \\ - \mu\left(2\zeta_2\rho\omega_1^2 u_2 - \frac{3\varepsilon_2\left(u_2^2+v_2^2\right)v_2}{4}\right) = 0. \end{gathered} \tag{9}$$

The system represented by (9) should be numerically solved. Then, the values of u_1, u_2, v_1 and v_2 can be calculated and the vibration amplitudes of the primary and secondary masses of the nonlinear DVA are obtained. The amplitude values are given by X_1 and X_2 according to the following equation:

$$X_i = \left(u_i^2 + v_i^2\right)^{0.5} \quad i = 1, 2 \tag{10}$$

3. Sensitivity Analysis of Structural Responses

In a mechanical system, the parameters of mass, stiffness, and damping establish the dependence with respect to a set of design parameters, which includes physical and geometrical characteristics and the parameters that are associated with the nonlinearities [24]. Such functional dependence can be expressed in a general form as follows:

$$\mathbf{r} = \mathbf{r}(\mathbf{M}(\mathbf{p}), \mathbf{C}(\mathbf{p}), \mathbf{K}(\mathbf{p})), \tag{11}$$

where $\mathbf{r}$ and $\mathbf{p}$ designate vectors of structural responses and design parameters, respectively.

(1) Initialize population with random solutions
(2) Evaluate fitness of the population
(3) While (stopping criterion not met)
(4) Select sites for neighborhood search
(5) Recruit bees for selected sites and evaluate fitnesses
(6) Select the fittest bee from each site
(7) Assign remaining bees to search randomly and evaluate their fitnesses
(8) End while

Algorithm 1: Basic step in the Bees Colony Algorithm [18].

The sensitivity of the structural responses with respect to a given parameter p_i evaluated for a given set of values of the design parameter p^0 is defined as the following partial derivative:

$$\frac{\partial \mathbf{r}}{\partial \mathbf{p}_i} = \lim_{\Delta \mathbf{p}_i \to 0} \left[R_1 + R_2 \right], \tag{12}$$

where

$$R_1 = \frac{\mathbf{r}\left(\mathbf{M}\left(\mathbf{p}_i^0 + \Delta\mathbf{p}_i\right), \mathbf{C}\left(\mathbf{p}_i^0 + \Delta\mathbf{p}_i\right), \mathbf{K}\left(\mathbf{p}_i^0 + \Delta\mathbf{p}_i\right)\right)}{\Delta\mathbf{p}_i},$$
$$R_2 = \frac{\mathbf{r}\left(\mathbf{M}\left(\mathbf{p}_i^0\right), \mathbf{C}\left(\mathbf{p}_i^0\right), \mathbf{K}\left(\mathbf{p}_i^0\right)\right)}{\Delta\mathbf{p}_i}, \tag{13}$$

where $\Delta\mathbf{p}_i$ is an arbitrary variation applied to the current value of parameter $\mathbf{p}_i^0$, while all other parameters remain unchanged. The sensitivity with respect to $\mathbf{p}_i$ can be estimated by finite differences by computing successively the responses corresponding to $\mathbf{p}_i = \mathbf{p}_i^0$ and $\mathbf{p}_i = \mathbf{p}_i^0 + \Delta\mathbf{p}_i$. Such procedure is an estimated approach enabling to calculate the sensitivity of the dynamic system responses with respect to small modifications introduced in the design parameters. Moreover, the results depend upon the choice of the value of the parameter increment $\Delta\mathbf{p}_i$. Another strategy consists in computing the analytical derivatives, if possible, of the structural responses with respect to the parameters of interest. However, this approach is not considered in the present paper.

4. Bioinspired Algorithms

Algorithms based on swarm intelligence principles are common in the literature due to the ability to find global solution in mono- and multiobjective contexts, different from algorithms based on gradients. According to Andrés and Lozano [25], swarm-based intelligence is artificial intelligence technique based on the study of collective behaviour in self-organizing systems, composed of a population of individuals, which takes effect between each other and environment. Although these systems do not have any central control of the individual behaviour, interaction between individuals and simple behaviour between them usually lead to detection of aggregate behaviour, which is typical for whole colony. This could by observed by ants, bees, birds, or bacteria in the nature. By inspiration of these colonies were develop algorithms called swarm-based intelligence and are successfully applied for solving complicated optimization problems [26]. In this contribution, three recent bioinspired optimization methods, Bees Colony Algorithm, Firefly Colony Algorithm, and Fish Swarm Algorithm, are considered as optimization strategies.

4.1. Bee Colony Algorithm. This optimization algorithm is based on the behavior of a colony of honey bees. The colony can extend itself over long distances and in multiple directions simultaneously to exploit a large number of food sources. In addition, the colony of honey bees presents as characteristic the capacity of memorization, learning, and transmission of information, thus forming the so-called swarm intelligence [27].

In a colony, the foraging process begins by scout bees being sent to search randomly for promising flower patches. When they return to the hive, those scout bees that found a patch which is rated above a certain quality threshold (measured as a combination of some constituents, such as sugar content) deposit their nectar or pollen and go to the waggle dance.

This dance is responsible for the transmission (colony communication) of information regarding a particular flower patch: the direction in which it will be found, its distance from the hive, and its quality rating (or fitness) [27]. The waggle dance enables the colony to evaluate the relative merit of different paths according to both the quality of the food they provide and the amount of energy needed to harvest it [28]. After waggle dancing, the dancer (scout bee) goes back to the flower patch with follower bees that were waiting inside the hive. More follower bees are sent to more promising patches. This allows the colony to gather food quickly and efficiently. While harvesting from a patch, the bees monitor its food level. This is necessary to decide upon the next waggle dance when they return to the hive [28]. If the patch is still good enough as a food source, then it will be advertised in the waggle dance, and more bees will be recruited to that source. In this context, Pham et al. [18] proposed an optimization algorithm inspired by the natural foraging behavior of honey bees (Bees Colony Algorithm (BCA)) as presented in Algorithm 1.

The BCA requires a number of parameters to be set, namely, the number of scout bees (n), number of sites selected for neighborhood search (out of n visited sites) (m), number of top-rated (elite) sites among m selected sites (e), number of bees recruited for the best e sites (nep), number of bees recruited for the other $(m - e)$ selected sites, neighborhood search (ngh), and the stopping criterion.

The BCA starts with the n scout bees being placed randomly in the search space. The fitnesses of the sites visited by the scout bees are evaluated in step 2.

In step 4, bees that have the highest fitnesses are chosen as selected bees, and sites visited by them are chosen for neighborhood search. Then, in steps 5 and 6, the algorithm conducts searches in the neighborhood of the selected sites, assigning more bees to search near to the best e sites. The bees can be chosen directly according to the fitnesses associated with the sites they are visiting.

Alternatively, the fitness values are used to determine the probability of the bees being selected. Searches in the neighborhood of the best e sites, which represent more promising solutions, are made more detailed by recruiting more bees to follow them than the other selected bees. Together with scouting, this differential recruitment is a key operation of the BCA.

However, in step 6, for each patch only the bee with the highest fitness will be selected to form the next bee population. In nature, there is no such restriction. This restriction is introduced here to reduce the number of points to be explored. In step 7, the remaining bees in the population are assigned randomly around the search space scouting for new potential solutions.

In the literature, various applications using this bioinspired approach can be found, such as modeling combinatorial optimization transportation engineering problems [21], engineering system design [15, 29], mathematical function optimization [18], transport problems [30], dynamic optimization [31], optimal control problems [32], and parameter estimation in control problems [16, 33] (http://mf.erciyes.edu.tr/abc/).

4.2. Firefly Colony Algorithm. The FCA is based on the characteristic of the bioluminescence of fireflies, insects notorious for their light emission. According to Yang [19], biology does not have a complete knowledge to determine all the utilities that firefly luminescence can bring to, but at least three functions have been identified: (i) as a communication tool and appeal to potential partners in reproduction, (ii) as a bait to lure potential prey for the firefly, and (iii) as a warning mechanism for potential predators reminding them that fireflies have a bitter taste.

In this way, the bioluminescent signals are known to serve as elements of courtship rituals (in most cases, the females are attracted by the light emitted by the males), methods of prey attraction, social orientation, or as a warning signal to predators [34].

It was idealized some of the flashing characteristics of fireflies so as to develop firefly-inspired algorithms. For simplicity the following three idealized rules are used [35]:

(1) all fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex;

(2) attractiveness is proportional to their brightness; thus, for any two flashing fireflies, the less brighter will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If there is no brighter one than a particular firefly, it will move randomly;

(3) the brightness of a firefly is affected or determined by the landscape of the objective function. For a maximization problem, the brightness can simply be proportional to the value of the objective function.

According to Yang [19], in the firefly algorithm there are two important issues: the variation of light intensity and the formulation of the attractiveness. For simplicity, it is always assumed that the attractiveness of a firefly is determined by its brightness, which in turn is associated with the encoded objective function.

This swarm intelligence optimization technique is based on the assumption that the solution of an optimization problem can be perceived as agent (firefly) which glows proportionally to its quality in a considered problem setting. Consequently, each brighter firefly attracts its partners (regardless of their sex), which makes the search space being explored more efficiently. The algorithm makes use of a synergic local search. Each member of the swarm explores the problem space taking into account results obtained by others, still applying its own randomized moves as well. The influence of other solutions is controlled by the attractiveness value [34].

According to Lukasik and Zak [34], the FCA is presented in the following. Consider a continuous constrained optimization problem where the task is to minimize the cost function $f(x)$ as follows

$$f(x^*) = \min_{x \in S} f(x). \tag{14}$$

Assume that there exists a swarm of m_a agents (fireflies) solving the above mentioned problem iteratively, and x_i represents a solution for a firefly i in algorithm's iteration k, whereas $f(x_i)$ denotes its cost. Initially, all fireflies are dislocated in S (randomly or employing some deterministic strategy). Each firefly has its distinctive attractiveness λ which implies how strong it attracts other members of the swarm. As the firefly attractiveness, one should select any monotonically decreasing function of the distance $r_j = d(x_i, x_j)$ to the chosen firefly j, for example, the exponential function

$$\lambda = \lambda_0 \exp\left(-\gamma r_j\right), \tag{15}$$

where λ_0 and γ are predetermined algorithm parameters: maximum attractiveness value and absorption coefficient, respectively. Furthermore, every member of the swarm is characterized by its light intensity I_i which can be directly expressed as an inverse of a cost function $f(x_i)$. To effectively explore considered search space S, it is assumed that each firefly i is changing its position iteratively taking into account two factors: attractiveness of other swarm members with higher light intensity, for example, $I_j > I_i$, for all $j = 1, \ldots, m_a$, $j \neq i$ which is varying across distance, and a fixed random step vector u_i. It should be noted as well that if no brighter firefly can be found, only such randomized step is being used.

Thus, moving at a given time step t of a firefly i toward a better firefly j is defined as

$$x_i^t = x_i^{t-1} + \lambda \left(x_j^{t-1} - x_i^{t-1} \right) + \alpha \left(\text{rand} - 0.5 \right), \quad (16)$$

where the second term on the right side of the equation inserts the factor of attractiveness, λ while the third term, governed by α parameter, governs the insertion of certain randomness in the path followed by the firefly, rand is a random number between 0 and 1.

In the literature, few works using the FCA can be found. In this context, is emphasized application in parameter estimation in control problems [16], continuous constrained optimization task [34], multimodal optimization [36], solution of singular optimal control problems [37], and economic emissions load dispatch problem [38].

4.3. Fish Swarm Algorithm. In the development of the FSA, based on fish swarm and observed in nature, the following characteristics are considered [20, 39]: (i) each fish represents a candidate solution of optimization problem; (ii) food density is related to an objective function to be optimized (in an optimization problem, the amount of food in a region is inversely proportional to the objective function value); (iii) the aquarium is the design space where the fish can be found.

As noted earlier, the fish weight at the swarm represents the accumulation of food (e.g., the objective function) received during the evolutionary process. In this case, the weight is an indicator of success [20, 39].

Basically, the FSA presents four operators classified into two classes: "food search" and "movement." Details on each of these operators are shown in the following.

4.3.1. Individual Movement Operator. This operator contributes to the movement (individual and collective) of fishes in the swarm. Each fish updates its position by using

$$x_i(t+1) = x_i(t) + \text{rand} \times s_{\text{ind}}, \quad (17)$$

where x_i is the final position of fish i at current generation, rand is a random generator, and s_{ind} is a weighted parameter.

4.3.2. Food Operator. The weight of each fish is a metaphor used to measure the success of food search. The higher the weight of a fish, the more likely this fish will be in a potentially interesting region in the design space.

According to Madeiro [39], the amount of food that a fish eats depends on the improvement in its objective function in the current generation and the greatest value considering the swarm. The weight is updated according to

$$W_i(t+1) = W_i(t) + \frac{\Delta f_i}{\max(\Delta f)}, \quad (18)$$

where $W_i(t)$ is the fish weight i at generation t, and Δf_i is the difference found in the objective function between the current position and the new position of fish i. It is important to emphasize that $\Delta f_i = 0$ for the fish in the same position.

4.3.3. Instinctive Collective Movement Operator. This operator is important for the individual movement of fish when $\Delta f_i \neq 0$. Thus, only the fish whose individual execution of the movement resulted in improvement of their fitness will influence the direction of motion of the swarm, resulting in instinctive collective movement. In this case, the resulting direction (I_d), calculated using the contribution of the directions taken by the fish, and the new position of the ith fish are given by

$$I_d(t) = \frac{\sum_{i=1}^{N} \Delta x_i \Delta f_i}{\sum_{i=1}^{N} \Delta f_i},$$
$$x_i(t+1) = x_i(t) + I_d(t). \quad (19)$$

It is worth mentioning that in the application of this operator, the direction chosen by a fish that located the largest amount of food exerts the greatest influence on the swarm. Therefore, the instinctive collective movement operator tends to guide the swarm in the direction of motion chosen by the fish who found the largest amount of food in its individual movement.

4.3.4. Noninstinctive Collective Movement Operator. As noted earlier, the fish weight is a good indication of success in the search for food. In this way, the swarm weight is increasing, this means that the search process is successful. So, the "radius" of the swarm should decrease to other regions to be explored. Otherwise, if the swarm weight remains constant, the radius should increase to allow the exploration of new regions.

For the swarm contraction, the centroid concept is used. This is obtained by means of an average position of all fish weighted with the respective fish weights, according to

$$B(t) = \frac{\sum_{i=1}^{N} x_i W_i(t)}{\sum_{i=1}^{N} W_i(t)}. \quad (20)$$

If the swarm weight remains constant in the current iteration, all fish update their positions by using

$$x(t+1) = x(t) - s_{\text{vol}} \times \text{rand} \times \frac{x(t) - B(t)}{d(x(t), B(t))}, \quad (21)$$

where d is a function that calculates the Euclidean distance between the centroid and the current position of fish, and s_{vol} is the step size used to control fish displacements.

In the literature, few works using the FSA can be found. In this context, parameter estimation in control problems [16], feed forward neural networks [40], parameter estimation in engineering systems [41], combinatorial optimization problem [42], augmented Lagrangian fish swarm based method for global optimization [43], forecasting stock indices using radial basis function neural networks optimized [44], and hybridization of the FSA with the Particle Swarm Algorithm to solve engineering systems are examples of successful applications [45].

Table 1: Nominal values of design variables.

Parameters	Nominal values
ε_1	0.001
ε_2	0.01
β	0.1
ζ_1	0.01
ζ_2	0.01
μ	0.05
ρ	1.0

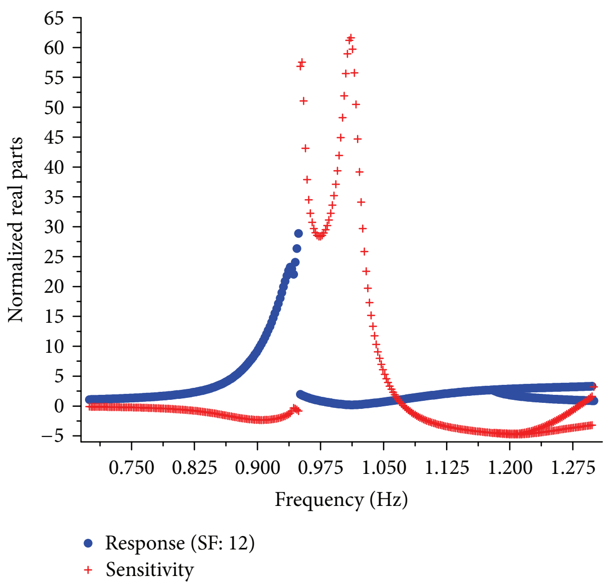

Figure 2: Sensitivity of $\mathbf{H}(\omega, \mathbf{p})$ with respect to ρ.

5. Results and Discussion

The following numerical example, studied in [6], is presented to illustrate the application of the proposed methodology to obtain the optimal design of a nDVA. As previously mentioned, the nDVA is used in the present contribution to represent a large class of nonlinear mechanical systems. Figure 1 depicts the test structure consisting of a primary mass attached to the ground by a nonlinear spring and coupled with an nDVA. The nominal values of the design parameters used to generate the dynamic responses of the nonlinear system are illustrated on Table 1. The computations performed consist in obtaining the driving point frequency responses associated to the displacement x_1.

5.1. Sensitivities of the Frequency Response with respect to Structural Parameters. To illustrate the computation procedure for the sensitivity of dynamic responses, numerical tests were performed by using the system configuration illustrated in Figure 1. As previously mentioned, the computations are devoted to obtaining the sensitivities of the driving point frequency responses, which are given by the elements of $\mathbf{H}(\omega, \mathbf{p})$.

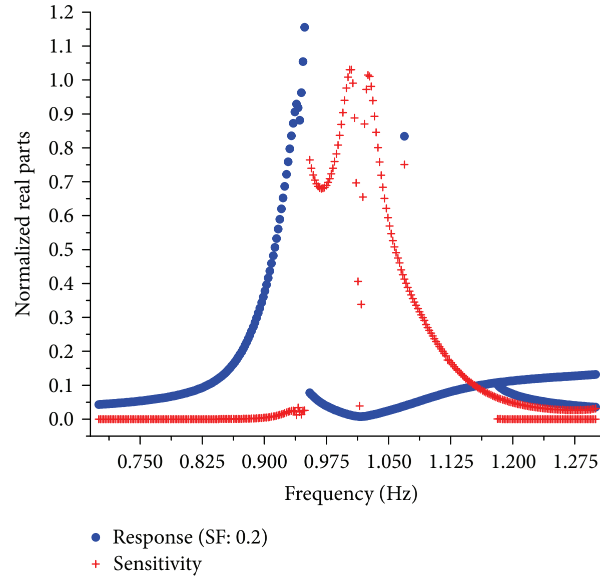

Figure 3: Sensitivity of $\mathbf{H}(\omega, \mathbf{p})$ with respect to ε_2.

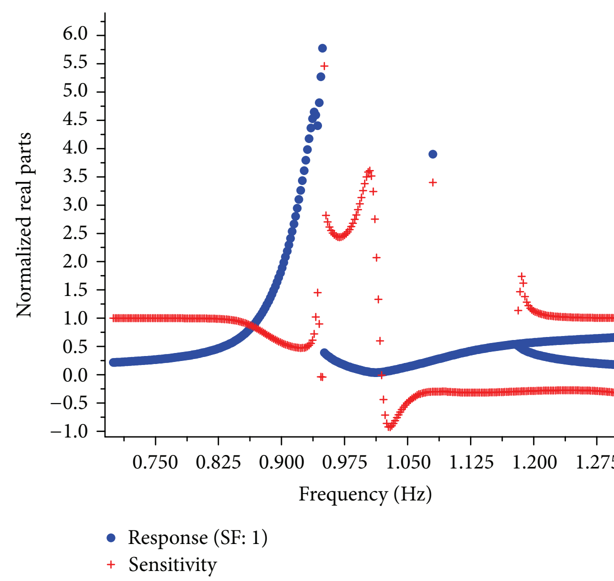

Figure 4: Sensitivity of $\mathbf{H}(\omega, \mathbf{p})$ with respect to β.

In this example, the normalized structural parameters ζ_1, ζ_2, ε_1, ε_2, β, μ, and ρ are considered as the design variables in the computation of the normalized sensitivities of the frequency responses with respect to a given parameter $\mathbf{p}$, $S_H^N(\omega, \mathbf{p})$. The normalized real parts of the approximated complex sensitivity functions calculated by finite differences (according to (17)) are shown in Figures 2, 3, 4, and 5, for which a variation of 20% of the nominal values of each design parameter was adopted. Also, in the same figures, the real parts of the frequency responses $\mathbf{H}(\omega, \mathbf{p})$, multiplied by

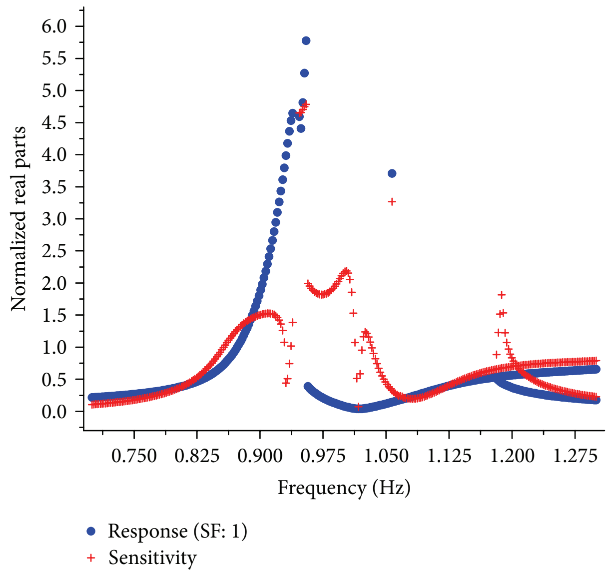

FIGURE 5: Sensitivity of $\mathbf{H}(\omega, \mathbf{p})$ with respect to μ.

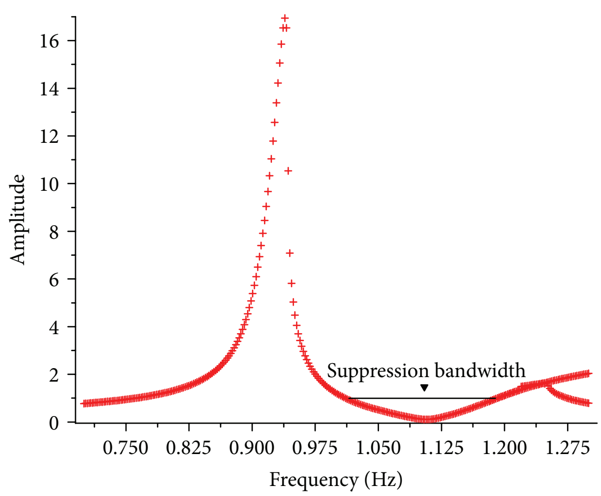

FIGURE 6: Representation of the objective function (maximize the suppression bandwidth).

convenient scale factors (SF), are shown. The sensitivity functions, denoted by $S_H^N(\omega, \mathbf{p})$, have been normalized according to the following scheme:

$$S_H^N(\omega, \mathbf{p}) = \left.\frac{\partial S_H(\omega, \mathbf{p})}{\partial \mathbf{p}}\right|_{(\omega, \mathbf{p}_0)} \frac{\mathbf{p}_0}{\mathbf{H}(\omega, \mathbf{p})}. \quad (22)$$

Based on the amplitudes and signs of the sensitivity functions, one can evaluate the degree of influence of the design variables upon the suppression bandwidth, in the frequency band of interest. In addition, the sensitivity analysis enables to decide among the design parameters those that will be retained in the optimization process. The parameters ζ_1, ζ_2, and ε_1 do not have a significant influence on the evaluation of the suppression bandwidth [6]. Consequently, these parameters are not considered as design variables in the optimization run. However, as shown in Figures 2, 3, 4, and 5, the degrees of influence of the parameters ε_2, μ, and ρ on the suppression bandwidth are significant and will be considered as design variables in the optimization process.

After having verified the influence of each design variable on the dynamic response of the nonlinear system, the interest now is to maximize the suppression bandwidth, as illustrated in Figure 6, using the bioinspired algorithms.

In these simulations, the following ranges are considered for the design parameters: $0.9 \leq \rho \leq 1.2$, $0.04 \leq \mu \leq 0.06$, $0.09 \leq \beta \leq 1.2$, and $0.009 \leq \varepsilon_2 \leq 0.012$.

In order to evaluate the performance of the three techniques proposed above (BCA, FCA, and FSA), the following parameters were used in the algorithms:

(1) BCA parameters: number of scout bees (10), number of bees recruited for the best e sites (5), number of bees recruited for the other selected sites (5), number of sites selected for neighborhood search (5), number of top-rated (elite) sites among m selected sites (5), neighborhood search (ngh) [10^{-3} 10^{-4} 10^{-6}], and generation number (50);

(2) FCA parameters: number of fireflies (15), maximum attractiveness value (0.9), absorption coefficient [0.7 0.9 1.0] and generation number (50);

(3) FSA parameters: number of fish (15), weighted parameter value (1), control fish displacements [10^{-1} 10^{-2} 10^{-3}], and generation number (50).

In order to examine the quality of the solution methodology considered, the Genetic Algorithm (GA) and the Particle Swarm Optimization (PSO) have been performed. The parameters used by GA and PSO are as follows:

(1) GA parameters: population size (15), crossover rate (0.8), mutation rate (0.01), and generation number (50);

(2) PSO parameters: population size (15), inertia weight (1), cognitive and social parameters (0.5), constriction factor (0.8), and generation number (50).

The stopping criterion used was the maximum number of iterations. Each case study was computed 20 times before calculating the average values. It should be emphasized that 1510 objective function evaluations for each algorithm are necessary.

In Table 2 the results (best (B), average (A) and worst (W)) obtained for the design of the nonlinear vibration absorber are presented.

This table shows that the three algorithms presented good estimates for the unknown parameters when compared with the GA and PSO. When the BCA is analyzed in terms of the neighborhood search parameter, the best result is obtained by using 10^{-3}, for example, a search region with smaller distances to exploit a large number of food sources. When the FCA is analyzed in terms of the absorption coefficient, the best result is found by using $\gamma = 1.0$, for example, thus emphasizing the local search. Finally, when the FSA is analyzed in terms of the control fish displacements, the best result is found by using $s_{\text{vol}} = 10^{-1}$.

Table 2: Results obtained by BCA, FCA, FSA, GA, and PSO to design the nonlinear vibration absorber (OF = objective function).

			ρ	μ	β	ε_2	OF
BCA	10^{-3}	B	1.165	0.053	0.110	0.017	0.236
		A	1.189	0.055	0.102	0.015	0.230
		W	1.027	0.058	0.091	0.011	0.218
	10^{-4}	B	1.099	0.056	0.103	0.010	0.230
		A	1.134	0.058	0.095	0.011	0.224
		W	0.916	0.045	0.103	0.011	0.224
	10^{-6}	B	1.134	0.058	0.095	0.011	0.224
		A	0.981	0.046	0.095	0.010	0.218
		W	0.961	0.050	0.105	0.011	0.218
FCA	0.7	B	1.158	0.054	0.110	0.017	0.230
		A	1.158	0.054	0.110	0.017	0.230
		W	1.157	0.053	0.110	0.017	0.228
	0.9	B	1.148	0.055	0.110	0.018	0.230
		A	1.148	0.055	0.110	0.018	0.230
		W	1.148	0.055	0.110	0.018	0.230
	1.0	B	1.156	0.056	0.111	0.017	0.232
		A	1.148	0.055	0.110	0.018	0.230
		W	1.151	0.057	0.109	0.015	0.214
FSA	10^{-1}	B	1.166	0.052	0.109	0.016	0.235
		A	1.191	0.050	0.101	0.016	0.232
		W	1.019	0.053	0.092	0.010	0.219
	10^{-2}	B	1.101	0.054	0.099	0.099	0.232
		A	1.135	0.060	0.094	0.011	0.225
		W	0.921	0.048	0.102	0.011	0.225
	10^{-3}	B	1.135	0.061	0.099	0.010	0.226
		A	0.993	0.049	0.096	0.010	0.219
		W	0.971	0.051	0.111	0.010	0.214
GA		B	1.101	0.054	0.099	0.098	0.233
		A	1.136	0.062	0.092	0.012	0.226
		W	0.932	0.044	0.110	0.012	0.228
PSO		B	1.101	0.053	0.099	0.099	0.233
		A	1.132	0.061	0.095	0.010	0.224
		W	0.918	0.049	0.103	0.011	0.225

6. Conclusions

In this work, the Bees Colony Algorithm, the Firefly Colony Algorithm, and the Fish Swarm Algorithm were proposed as alternative techniques to obtain the optimal design of a nonlinear mechanical system. For illustration purposes they were applied in the design of a nonlinear dynamic vibration absorber. The system nonlinearity was introduced in the springs that connect the primary mass to the ground and the absorber to the primary mass, respectively.

As observed in Table 2, the algorithms led to satisfactory results in terms of the effectiveness of the optimal nDVA configuration and the number of objective function evaluations, when compared with GA and PSO strategies. However, the results obtained by the algorithms need yet to be better analyzed, so that more definitive conclusions can be drawn, for example, new mechanisms for diversity exploration should be studied.

The choice of the design variables is based on previous knowledge regarding their sensitivities with respect to the suppression bandwidth. It is worth mentioning that these parameters are directly associated with the effectiveness of the nDVA.

In terms of the system resolution, the equations of motion of the nonlinear two d.o.f. system were numerically integrated by using the so-called average method that provides an approximate solution to nonlinear dynamic problems. The nonlinear algebraic equations obtained were solved numerically enabling to determine the roots of the nonlinear algebraic equations.

It is worth mentioning that the nonlinearity factor is an important parameter to be investigated during the design procedure of nDVAs, due to its contribution to the reduction of the vibration level. However, care must be taken with high values of nonlinearity because of the instabilities introduced in the system. This point motivates an important aspect regarding the proposed methodology: obtaining the optimal spring nonlinear coefficient that guarantees the best stable solution for a given system.

Finally, the optimization techniques used in this paper can be successfully applied in the design of a great number of nonlinear mechanical systems.

As further work, we intend to extend the algorithms to the multiobjective context and assess the sensitivity of the parameters with respect to the effectiveness of the solution.

Nomenclature

B: Weight matrix
BCA: Bees Colony Algorithm
BiOM: Bioinspired Optimization Methods
C: Damping matrix
c_i: Damping coefficients
d: Euclidean distance
DVAs: Dynamic vibration absorbers
e: Number of top-rated sites
f: Objective function
$\mathbf{f}$: Force vector
F_1: Force applied to the primary system
FCA: Firefly Colony Algorithm
FSA: Fish Swarm Algorithm
$\mathbf{H}(\omega, \mathbf{p})$: Sensitivities of the driving point frequency responses
I: Light intensity
I_d: Fishes movement direction
K: Stiffness matrix
K_i: Linear coefficients of the springs
k_i^{nl}: Nonlinear coefficients of the springs
M: Mass matrix
m: Number of sites selected for neighborhood search
m_a: Number of fireflies
m_1: Primary mass
m_2: Secondary mass
n: Number of scout bees
nep: Number of bees recruited for the best e sites
ngh: Neighborhood search
$\mathbf{p}$: Design parameters
P: Normalized reference value
$\mathbf{r}$: Vectors of structural responses
r_1: Constitutive forces of the springs
R_i: Auxiliary variables vector
r_j: Distance function
S: Design space
S_H^N: Sensitivity function
s_{ind}: Weighted parameter
s_{vol}: Control fish displacements
$\mathbf{u}$: Auxiliary variables vector
$\mathbf{v}$: Auxiliary variables vector
x_1: Displacement of the primary system with respect to the ground
x_2: Displacement of the DVA with respect to the primary system
x_i: Design variables
x_c: Displacement reference value
X_i: Vibration amplitudes
y_i: Normalized displacements
W: Fish weight
α: Insertion of randomness
β: Normalized force parameter
δ_i: Normalized reference value
η_i: Normalized reference value
λ: Attractiveness
λ_0: Maximal attractiveness
ω: Normalized reference value
Ω: Normalized frequency value
ξ_i: Normalized coefficient of the damping
μ: Normalized mass ratio
ρ: Normalized frequency ratio
τ: Dimensionless time
γ: Absorption coefficient
ε_i: Normalized nonlinear coefficient of the springs.

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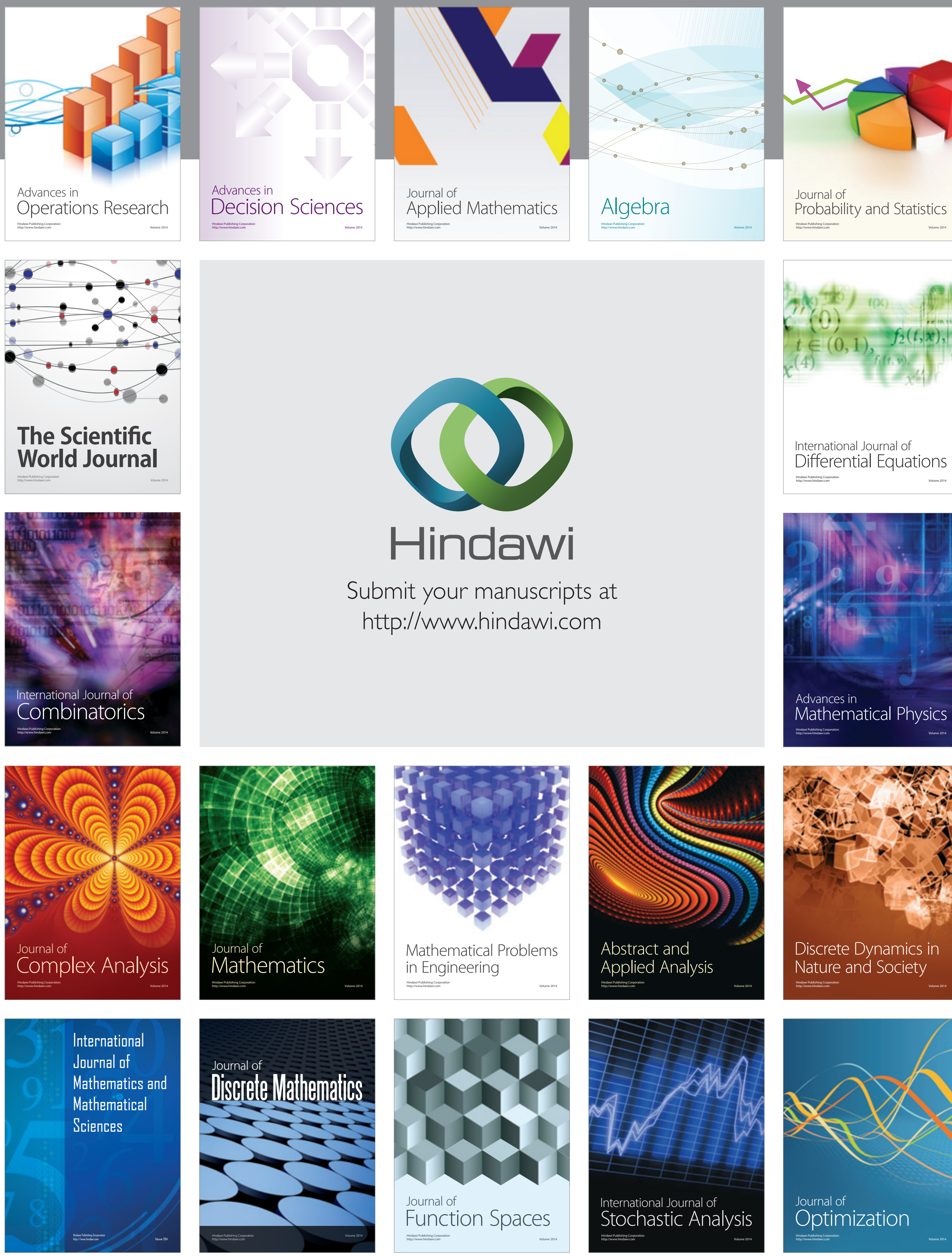
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