Universidade Federal do Goiás
Faculdade de Administração, Contabilidade e Economia - FACE
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Third Degree Price Discrimination with Externality in Cournot Oligopoly

Anna Eloyr Silveira Vilasboas

Goiânia - GO, 06 de maio de 2020

UNIVERSIDADE FEDERAL DE GOIÁS
FACULDADE DE ADMINISTRAÇÃO, CIÊNCIAS CONTÁBEIS E CIÊNCIAS ECONÔMICAS

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ATA DE DEFESA DE DISSERTAÇÃO

Ata nº 2 da sessão de Defesa de Dissertação de Anna Eloyr Silveira Vilasboas, que confere o título de Mestra em Economia, na área de concentração em Economia Aplicada.

Aos seis dias do mês de maio do ano de dois e vinte, a partir das 10 horas e 30 minutos, por meio de videoconferência, realizou-se a sessão pública de Defesa de Dissertação intitulada "Third Degree Price Discrimination with Externality in Cournot Oligopoly". Os trabalhos foram instalados pelo Orientador, Professor Doutor Francisco Bruno de Lima Holanda (PPGECON/FACE/UFG), com a participação dos demais membros da Banca Examinadora: Professor Doutor Wilfredo Fernando Leiva Maldonado (PPGECON/FACE/UFG), membro titular interno e Professor Doutor Juan Pablo Gama Torres (UFMG), membro titular externo. Durante a arguição os membros da banca **fizeram** sugestões **em documentos enviados por e-mail à candidata**. A Banca Examinadora reuniu-se em sessão secreta a fim de concluir o julgamento da Dissertação, tendo sido a candidata **aprovada** pelos seus membros. Proclamados os resultados pelo Professor Doutor Francisco Bruno de Lima Holanda, Presidente da Banca Examinadora, foram encerrados os trabalhos e, para constar, lavrou-se a presente ata que é assinada pelos Membros da Banca Examinadora, aos seis dias do mês de maio do ano de dois mil e vinte.

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"We do not remember days,
we remember moments."
(Cesare Pavese)

Resumo

As empresas geralmente consideram rentável segmentar clientes de acordo com a sensibilidade à demanda e discriminar preços de acordo com essa divisão. Em algumas situações, a heterogeneidade do consumidor pode ser diretamente observada, e uma empresa pode basear seus preços em característica, como gênero e idade. Neste contexto, a presente dissertação analisa a aplicação da discriminação de preço de terceiro grau com base no atributo gênero em mercados oligopolistas com de externalidades de consumo. Assumindo função de demanda linear, investigamos alterações no bem-estar social (a soma dos excedentes dos consumidores e do lucro do produtor) de acordo com a mudança de regime de preços uniformes para o discriminado. Demonstra-se que o bem-estar social e o excedente de consumidor masculino e do produtor não são afetados pela mudança de regime de preços. No entanto, o excedente de consumidor das mulheres aumenta com o preço discriminado.

Palavras-chaves: Oligopólio. Modelo de Cournot. Discriminação de preços de terceiro grau. Externalidade

Abstract

Firms often find it profitable to segment customers according to their demand sensitivity and to price discriminate accordingly. In some settings, consumer heterogeneity can be directly observed, and a firm can base its pricing upon contractible consumer characteristics. In this context, the presente dissertation analyses olipolistic third-degree price discrimination in the presence of consumption externalities within female and male markets. Assuming linear inverse demands, we investigate an associated change in social welfare (the sum of the consumers' surpluses and the producer's profit) with the regime change from uniform pricing to price discrimination when both markets are open under either regime. It is shown that social welfare and men's consumer surlus does not is unaffected by the regime change. However, women's consumer surplus increases with price discriminated.

Keywords: Oligopoly. Cournot model. Third-degree price discrimination. Social welfare. Consumption externality

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Introduction

According to technical note nº 2/2017 of the National Consumer Secretariat, institution linked to Justice Ministry, the leisure sector establishments, like bars and nightclubs, could not charge different prices for men and women from 2017 in Brazil. The justification given in the note was that differentiated charge affronts the dignity of the human person and the principle of isonomy. However, does this attitude increase women's well-being?

In economy, this kind of strategy is called price discrimination, which arises naturally in the theory of monopoly and oligopoly. Whenever a good is sold at a price in excess of its marginal cost, there is an incentive to engage in price discrimination. For to say that price is in excess of marginal cost is to say that there is someone who is willing to pay more than the cost of production for an extra unit of the good. Lowering the price to all consumers may well be unprofitable, but lowering the price to the marginal consumer alone will likely be profitable. (Varian, 1987)

In order to lower the price only to the marginal consumer, or more generally to some specific class of consumers, the firm must have a way to sort consumers. The easiest case is where the firm can explicitly sort consumers with respect to some exogenous category such as age. A more complex analysis is necessary when the firm must price discriminate on the basis of some endogenous category such as time of purchase. (Varian, 1987)

In an ideal business world with frictionless market, companies would be able to eliminate all consumer surplus, which is an economic measurement of consumer benefits, through first-degree price discrimination. This type of pricing strategy takes place when businesses can accurately determine what each customer is willing to pay for a specific product or service and selling that good or service for that exact price. (PINDYCK; RUBINFELD et al., 2013)

In second-degree price discrimination, the ability to gather information on every potential buyer is not present. Instead, companies price products or services differently based on the preferences of various groups of consumers. Most often, businesses apply second-degree price discrimination through quantity discounts. Customers who buy in bulk receive special offers not granted to those who buy a single product.(PINDYCK; RUBINFELD et al., 2013)

Third-degree price discrimination occurs when companies price products and services differently based on the unique demographics of subsets of its consumer base, such as students, military personnel, or seniors. Companies can understand the broad characteristics of consumers more easily than the buying preferences of individual buyers.

This type of pricing strategy is often seen in movie theater ticket sales, admission prices to amusement parks, or restaurant offers, which is the situation analyzed in the present work. Consumer groups that may, otherwise, not be able or willing to purchase a product due to their lower income are captured by this pricing strategy, increasing company profits.(PINDYCK; RUBINFELD et al., 2013)

Three conditions are necessary in order for price discrimination to be a viable solution to a firm's pricing problem. First, the firm must have some market power. Second, the firm must have the ability to sort consumers. And third, the firm must be able to prevent resale. (Varian, 1987)

The effect of third-degree price discrimination on prices and welfare has long been of interest to economists. However, from the seminal work of Robinson (1933) to more recent analyses, most research on this subject has addressed monopoly price discrimination. Despite more recent interest in competetitive price discrimination, only Holmes (1989) extends the traditional literature's analytical approach to third-degree price discrimination in environments of imperfect competition. (CORTS, 1998)

In the context of price discrimination and imperfect competition, we can find the situation of the female public in the market of night parties, where the practice of price discrimination by gender is recurrent. The owners of the establishments claim that this practice is adopted as a legitimate business strategy of balancing male and female audiences. According to entrepreneurs, a number of socioeconomic and cultural factors hampers women's access to leisure and entertainment. In contrast, representatives of consumer protection agencies argue that men and women are equal in rights and obligations under the Constitution.

In the face of this impasse, the present work seeks to analyze whether the prohibition of price discrimination increases women and men's social welfare through Cournot's oligopoly model. According to this theory, each company decides how much to produce and all firms must make their decisions simultaneously. Each one considers the level of production of its competitors to be fixed when deciding.

Furthermore, in the entertainment market, it is possible to notice that the female public has a clear influence on men's demand for ticket to particular party. In those situations, where a consumer group influences the other's demand curve, a kind of externality arises over this function. Thus, the more female participants, the greater the men's demand for tickets. To analyze role of externality in third degree price discrimination strategy and its impact on social welfare, the present work will use Cournot's model.

Therefore, this master's dissertation is divided into four chapters beyond this introduction. In the chapter 1, we describe the three types of price discrimination. In the next chapter, 2, literature review of the price discrimination strategy and its application

in Cournot oligopoly is performed. The chapter 3 presents the theoretical model that will be used. The chapter 4 brings comparative welfare analyzes between uniform prices and those with differentiation. Finally, there is the conclusion of the study.

1 Price Discrimination

Price discrimination occurs when a firm charges different prices for different units of output, and the price difference does not reflect a difference in marginal cost.For this to be profitable, the firm must have market power, otherwise the price would be exogenous to the firm. When selling the same product at different prices to different groups of consumers, a firm must also be able to keep consumers from trading among themselves.(TREMBLAY; TREMBLAY, 2012)

Furthermore, price discrimination is one of the most prevalent forms of marketing practices. One may occasionally doubt whether firms really engage in some of the kinds of sophisticated strategic reasoning economists are fond of examining, but there can be no doubt that firms are well aware of the benefits of price discrimination.(VARIAN, 1987)

The traditional classification of the forms of price discrimination is due to Pigou (1929). First-degree, or perfect price discrimination, involves the seller charging a different price for each unit of the good in such a way that the price charged for each unit is equal to the maximum willingness to pay for that unit.

Second-degree price discrimination, or nonlinear pricing, occurs when prices differ depending on the number of units of the good bought, but not across consumers. That is, each consumer faces the same price schedule, but the schedule involves different prices for different amounts of the good purchased. Quantity discounts or premia are the obvious examples.

Third-degree price discrimination means that different purchasers are charged different prices, but each purchaser pays a constant amount for each unit of the good bought. This is perhaps the most common form of price discrimination; examples are student discounts, or charging different prices on different days of the week.(VARIAN, 1987)

We will follow Pigou's classification in this chapter, discussing the forms of price discrimination in the order in which he suggested them.

1.1 First-degree price discrimination

First-degree or perfect price discrimination occurs when a monopolist sells each unit of output at the maximum price that each consumer is willing to pay for it. Recall that this price is called the consumer's demand price or reservation price. Although perfect price discrimination is impractical and not encountered in the real world, the model is rather simple and provides insights into a firm's motivation to price discriminate. It is

said to be perfect because we will see that it converts all consumer surplus into firm profit.(TREMBLAY; TREMBLAY, 2012)

Until recently, it has been a widely shared belief that first-degree price discrimination was little more than a theoretical curiosity. It is thus not surprising that, in contrast to second and third-degree price discrimination, which have been abundantly discussed (Schmalensee 1981; Bester and Petrakis 1996; Hamilton and Slutsky 2004; Liu and Serfes 2005; Galera and Zaratiegui 2006; Malueg and Snyder 2006), the literature devoted to first-degree price discrimination is comparatively scarce.(RAYNA; DARLINGTON; STRIUKOVA, 2015)

There are several reasons for first degree price discrimination' has received little attention in economic literature, primarily because of the contention that models describing such behavior do not reflect "real world" behavior. A second, less compelling argument, is that the economic solution to first degree price discrimination is usually considered to be so simple as to be uninteresting.(WHITE; WALKER, 1973)

In third place, in reality, we do not observe perfect price discrimination. In a market with many customers, the monopolist will not know every consumer's demand price, and it may be impractical to charge each one a different price even if this information were known with certainty. Nevertheless, the model provides a simple illustration of how price discrimination can increase firm profits and reduce allocative inefficiency, although at the expense of consumer welfare.(TREMBLAY; TREMBLAY, 2012)

Meantime, the advent of the internet (and of the digital economy, in general) has brought first-degree price discrimination back into focus. The first reason for that is that digital technologies make it possible, for the first time, to accurately (if not fully) observe the behavior of consumers, thereby enabling to estimate (or even fully reveal) their valuation of digital goods or services. Besides the early 'proof of concept' delivered by Amazon in 2000, which was based on information collected through internet cookies, technologies such as Digital Rights Management systems (embedded in music, video files or games) provide firms with abundant information about consumers' consumption patterns.

More recently, the advent of big data, which enables to obtain an even more accurate picture by combining the data collected from a particular consumer with a large quantity of data of similar customers (e.g. Apple's "Genius" functionality in iTunes), has brought companies even closer to a perfect valuation of willingness to pay. Hence, first-degree price discrimination is, more than ever, closer to becoming a reality.(RAYNA; DARLINGTON; STRIUKOVA, 2015)

1.2 Second-degree price discrimination

With second-degree or nonlinear price discrimination, the monopolist charges different prices for different quantities sold, but each consumer faces the same price structure. In other words, each consumer who purchases the same quantity pays the same price. The most common example is quantity or bulk discounting that is found at most supermarkets.The paradigm case of this form of price discrimination is quantity discounts.(TREMBLAY; TREMBLAY, 2012)

You can see why this is sometimes called nonlinear pricing. Cost asymmetries can cause firms to offer quantity discounts. Shipping costs provide one example, where the per-unit cost of packaging and shipping may be lower on large orders. In addition, firms may use quantity discounts to increase total sales, which lowers long-run average production costs when scale economics are present.(TREMBLAY; TREMBLAY, 2012)

Another motivation for using nonlinear pricing is that it can be a useful, although imperfect, way around the information problem associated with perfect price discrimination. The problem is that if the firm cannot distinguish consumers with high from consumers with low demand prices, the firm can set up a menu of price–quantity options that will induce consumers to choose (or self select) the package that extracts the most surplus from each type of consumer.(TREMBLAY; TREMBLAY, 2012)

Curiously, the determination of optimal nonlinear prices was not carefully examined until Spence (1977). According to the author, the use of quantity dependent prices for a single commodity can increase revenues, redistribute income, or to change the pattern of consumption of the good across people with different preferences. The monopolist tends to employ quantity discounts to raise revenues because quantity discounts keep the average price to the high value users high while lowering the marginal prices to encourage consumption.

Spence, however, does not pay explicit attention to the key role played by constraints on optimal pricing schedules. Roberts (1979) considers upward price discontinuities which lead to bunching of consumers at a given level of consumption, but not downward price discontinuities, which lead to regions in which no consumer chooses to consume (gaps). Bunching or gaps may appear in the optimal price schedule even when underlying demand and cost functions are smooth and well behaved. The prevalence of discontinuous pricing schedules (e.g. the block pricing structures of public utilities) argues that these questions are of practical as well as theoretical interest.

Katz (1983) has examined the welfare and output effects of nonlinear pricing. He shows that in general the monopolist may produce too little or too much output as compared with the social optimum, but when the non-crossing condition[1] is satisfied, the

[1] The property implies that the demand function for the high-demand consumer is always greater than

monopolist will typically restrict total output. In general, total welfare will be positively associated with total output so that changes in output may serve as appropriate indicators for changes in welfare.

1.3 Third-degree price discrimination

Third-degree or segmented price discrimination occurs when a monopolist divides or segments consumers into two or more groups and charges each group a different price. In this case, price discrimination is based on observable customer characteristics. This is the most common form of price discrimination and is found throughout the service sector. For instance, price reductions are frequently offered to women at nightclubs, senior citizens at hotels, and students at amusement parks. (TREMBLAY; TREMBLAY, 2012)

To see how this form of discrimination can be more profitable than charging a single price, we want to use an example presented in Tremblay e Tremblay (2012). Comparing the firm profits when the monopolist charges a single price with when it price discriminates. To simplify the problem we assume that a monopoly ski resort price discriminates between just two groups: adults (A) and children (C). Adult inverse demand is $p_A = 12 - Q_A$, and child inverse demand is $p_C = 6 - Q_C$, where output (Q) is measured in thousands of adult or children patrons per day. Long-run average cost (AC) and marginal cost (MC) equal 2.

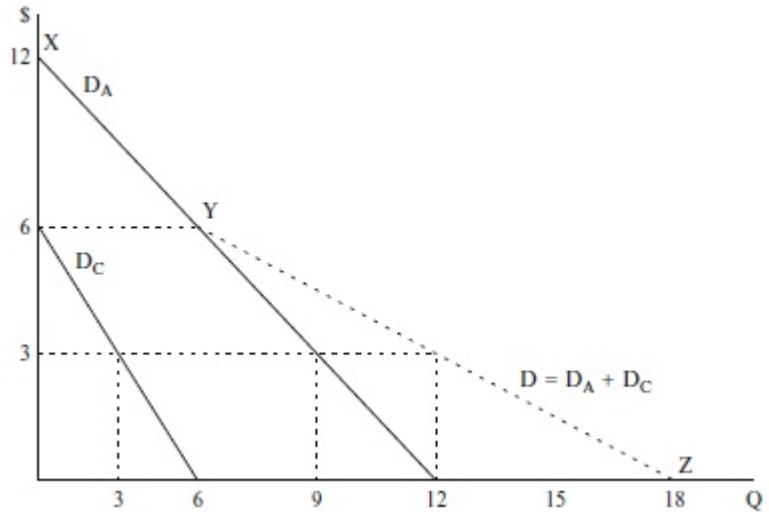
First, we consider the case where the monopolist charges a single price. To see this, we must derive the market demand function, which is adult plus children demand. Geometrically, the market demand (D) is the horizontal sum of adult demand (DA) and child demand (DC) in Figure 1. That is, when the price exceeds 6, only adults purchase tickets and the market demand corresponds to the adult demand, segment XY. At a lower price, such as 3, the market demand is the sum of child demand (3) and adult demand (6), which is 12. Thus, when the price falls below 6, the market demand corresponds to segment YZ.

$$p = \begin{cases} 12 - Q, & \text{for } p \geq 6 \\ 18 - 2Q, & \text{for } p < 6 \end{cases} \tag{1.1}$$

Because demand has a kink, marginal revenue (MR) is disjoint at the output level that corresponds to point Y, as diagrammed in Figure 2. To maximize profit, the monopolist will equate marginal revenue and marginal cost. In this example, however, marginal revenue crosses marginal cost in two places, E1 and E2. When this happens, the monopolist must compare the profits for each case and choose the one that earns the most profit (π). At E1, optimal values are $Q^* = 5$, $p^* = 7$, and $\pi^* = 25$. At E2, optimal values

the demand function for the low-demand consumer.

Figure 1 – Market demand (D) as the horizontal sum of adult demand (DA) and child demand (DC)



are $Q^* = 7$, $p^* = 5.5$, and $\pi^* = 24.5$. Thus, E1 is the optimum. Notice that only adults will attend the resort at this price.

Next, we calculate the outcome when the monopolist price discriminates between adults and children. We assume that the monopolist can keep the markets separate. In this case, the monopolist will treat each group as if they belong to a separate market and set the monopoly price in each. In the adult market, the marginal revenue is $12 - 2q$. Equating marginal revenue with marginal cost (which is 2) gives the profit-maximizing outcome: $q_A^* = 5$, $p_A^* = 7$, and $\pi_A^* = 25$.

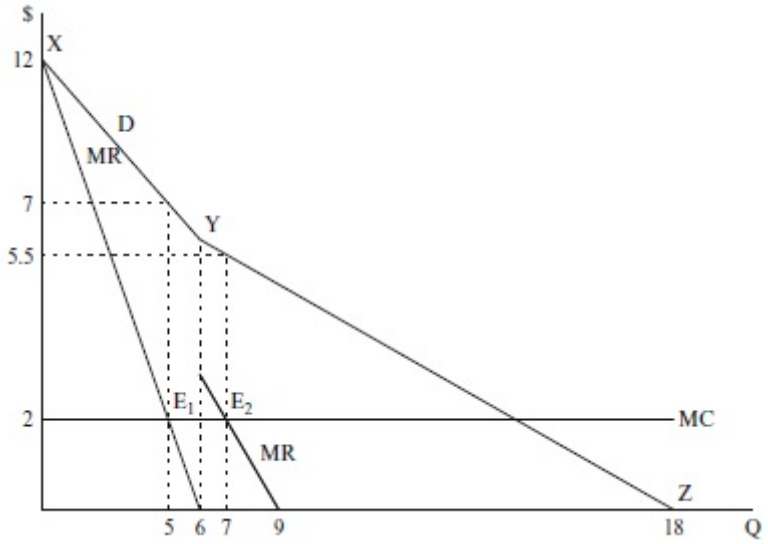
In the children's market where the marginal revenue is $6-2q$, the monopoly outcome is $q_C^* = 2$, $p_C^* = 4$, and $\pi_C^* = 4$. This is illustrated in Figure 3, where the demand and marginal revenue functions for children are reflected over the vertical axis and connected with the diagram of demand and marginal cost functions for adults.

Compared to the case with a single price, price discrimination is better for both consumers and producers. With price discrimination, profits increase from 25 to 29. Consumer surplus for adults remains unchanged because the adult price is unchanged [1]. No children enter the resort when there is a single price of 7. But with price discrimination, the children's price is 4, and 2 children enter the resort, increasing the consumer surplus for children from 0 to 2. Thus, in this example price discrimination raises both producer and consumer surplus.

Pigou (1929) in his comparison of output under discrimination of the third degree with that of simple monopoly, also distinguished "three principal cases" in which a monopolist with two markets and the ability to discriminate might find himself: Case I, in

[1] This assumes that the utility of adults is not raised or lowered by having children present.

Figure 2 – Market demand, marginal revenue and marginal cost



which conditions are such that under simple monopoly both markets are served; Case II, in which conditions are such that under simple monopoly only one of the markets is served; and Case III, in which conditions are such that under simple monopoly neither market is served. In his analysis Pigou further subdivided Case II into two distinct categories, where only one market is served under discrimination (Case II-A), and the other where both markets are served under discrimination (Case II-B).

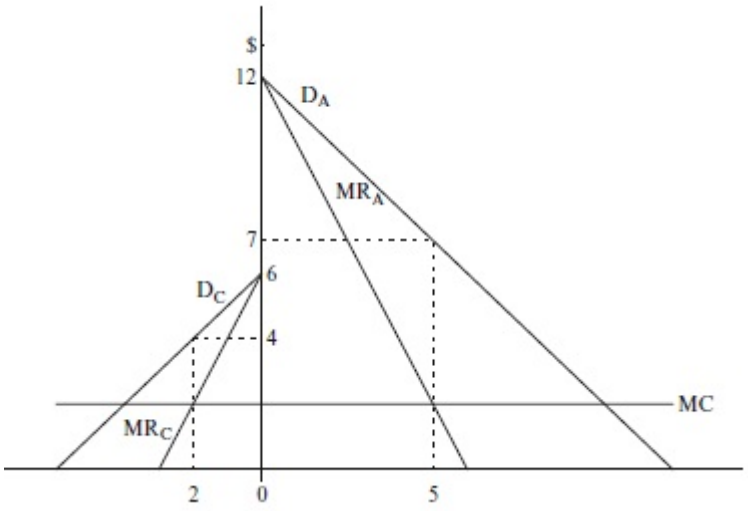
According to Varian (1987), much of the discussion about third-degree price discrimination has to do with the welfare effects of allowing this form of price discrimination. Would we generally expect consumer plus producer surplus to be higher or lower when third-degree price discrimination is present than when it is not?

The effect on social welfare of third-degree price discrimination was first investigated by Robinson (1933), but subsequent analysis has been focused almost exclusively on those conditions under which both markets are served by the simple monopolist, Case I.

Battalio e Ekelund (1972) tried to bridge this gap by focusing on those situations where one market is served with a single monopoly price, but more than one are supplied under discrimination.The introduction of discrimination may have important effects upon total monopoly output and upon the distribution of output between markets. These effects are obtrusive in all Case II-B situations. Whether or not simple monopoly output will be produced in Cases I, II-A, II-B, or III is, of course, solely dependent upon the parameters of the cost and revenue functions. Our analysis indicates that, for some of these values, output and price under the simple monopoly structure posited may exhibit an element of discreteness which is carried over to the discriminating monopoly.

Schmalensee et al. (1980) has re-examined the question introduced by Robinson and presented several new results. In particular, he noted that a necessary condition for

Figure 3 – Third-degree price discrimination among adults (A) and children (C)



price discrimination to increase social welfare - defined as consumer plus producer surplus - is that output increase. Schmalensee established this 'result only in the case of independent demands and constant marginal costs. However, it turns out to be true in much more general circumstances.

2 Third-Degree Price Discrimination in Oligopoly

Analysis of third-degree price discrimination has a long history. Pigou (1929) and Robinson (1933) are the classic contributions in this area. Traditionally price discrimination was analysed for industries characterized by monopoly. Varian (1989) provides a succinct summary of this literature. In recent years, there have been papers such as Holmes (1989) and Corts (1998) analysing the effects of third-degree price discrimination in an oligopolistic market structure. Holmes is a pioneer in this respect.

According to Kumar e Kutlu (2016), the differentiated products framework is dominant in the price competition setting and the homogenous goods framework is dominant in the quantity competition setting.The common approach in the literature on price discrimination in oligopoly follows price competition approach.

In the present chapter of dissertation, we concentrate on the homogenous goods framework where the valuations of customers are represented by a function of some characteristics. Based on those characteristics firms optimally segment the customers in various bins. The closest paper to ours that considers price discrimination in quantity competition is Hazledine (2006). For the linear demand case, Hazledine analyzes price discrimination in the Cournot framework where firms decide on quantities to sell in various bins.

2.1 Symmetric oligopoly model

Holmes (1989) shows that a firm's price elasticity of demand in a market can be expressed as the sum of two parts: i) the industry-demand elasticity and the cross-price elasticity. The first measures the tendency of consumers in a market to stay home when the price goes up; the second the tendency to switch suppliers. He concluded that, in the noncooperative oligopoly case, equilibrium price differentials can be accounted for both by differences in cross-price elasticity as well as differences in industry-demand elasticity.

According to Holmes, total industry output when the firms can discriminate between two markets is compared to what it would be if discrimination were impossible. Which regime has larger output depends on the sum of the "adjusted concavity condition" and the "elasticity-ratio condition". The first predominates when the rival products are poor substitutes and the firms have approximate monopoly power, while the second predominates at the other extreme where the products are close substitutes and the firms have almost

no market power.

The adjusted-concavity condition compares the relative curvature of demands in the two markets and was originally discovered by Robinson (1933) in her analyses of monopoly price discrimination, which will be more detailed in the chapter about welfare analysis. The elasticity condition compares the relative difference in the industry-demand elasticity between the two markets with the relative difference in cross-price elasticity. According to the elasticity-ratio condition, a discriminatory discount due to high industry elasticity of demand would be more likely to increase total output than one due to high cross-price elasticity.

The effects of discrimination on profit is also discussed in Holmes (1989). The distinction made between industry-demand elasticity and cross price elasticity proves useful in finding a situation in which discrimination can reduce profit levels.

In short, Holmes demonstrates that in a symmetric oligopoly model the effects of price discrimination are quite similar to its effects in the monopoly case; in particular, the uniform price necessarily lies between the discriminatory prices, so that permitting price discrimination leads to higher prices for some consumers and lower prices for others.

In his paper, Dastidar (2006) follow Holmes (1989) to focus on a completely symmetric duopoly. Starting from a basic multimarket, Dastidar provided sufficient conditions for output, profit and welfare to be higher (or lower) in the third-degree price discrimination under duopoly compared with a uniform price regime. The author shows that output and profit effects under duopoly price discrimination are very different from that under monopoly price discrimination. However, the welfare effects are the same.We

2.2 Assymetric olipoly model

In contrast, Corts (1998) investigate the price and welfare effects of third-degree price discrimination in a general differentiated-goods oligopoly model, relaxing the assumptions of symmetric demand made by Holmes. He shows that unambiguous price and welfare effects may arise when firms differ in their ranking of consumer groups by their demand elasticities at a given rival's price.

Holding rivals' prices fixed, every firm has an incentive to raise price to its "strong market" and to lower price to its "weak" market. In an asymmetric model, all firms need not rank the same group as the strong market. As a result, some firm may have an incentive to lower price to every group when permitted to price discriminate; if this elicits a strong enough competitive response, prices may fall for every group and consumer welfare may unambiguously increase for all consumers. While Corts (1998) focus on the case in which third-degree price discrimination by imperfectly competitive firms leads to all-out

competition, characterized by lower prices for all consumers and unambiguous consumer welfare gains, it is also possible that all prices rise when firms are permitted to discriminate, leading to unambiguous consumer welfare losses.

When a monopolist discriminates, it necessarily earns higher profits than under uniform pricing, as it solves the same profit-maximization problem with fewer constraints. But outside of the monopolistic case, price discrimination may lead to lower profits for firms; as a result, imperfectly competitive firms may wish to avoid price discrimination and the all-out competition that may ensue, perhaps through commitments to uniform pricing.

Corts (1998) generalizes Holmes' model to allow for situations where the consumer groups have asymmetric rankings of the firms' products. In this case, the weak market for one firm is the strong market for the other, and vice versa. This opens the possibility that price discrimination yields prices that are lower for both groups than the uniform price, with corresponding lower profits for everyone. Hence, firms would like to commit to a uniform pricing strategy if possible, and unilaterally employing such a strategy may even be profitable if pre-commitment is credible (such as a "no haggle" policy or an "everyday low price" program). From the firms' point of view, the all-out competitive outcome exhibits too much price discrimination.

2.3 Cournot olipoly model with third-degree price discrimination

However, none of this research has worked with Cournot's model. The first work about quantity competition was Hazledine (2006). He extends the standard Cournot–Nash oligopoly model to the case where firms charge groups of customers different prices for the same product, based on their willingness to pay. That is, he relaxs the uniformprice assumption of the standard model, though only in the simplest context of linearity, symmetric firms and a homogeneous product. The main finding is that in any n-firm Cournot–Nash oligopoly, the average price charged is invariant to the extent of price discrimination, so that the standard single-price model does not mislead in this respect, although it will always under-predict total industry output.

To a linear approximation, extending the symmetric Cournot–Nash oligopoly model to allow for firms charging a range of prices based on customers' willingness to pay does not change the predicted average price or revenue in equilibrium, though total industry output – and, therefore, profits – will increase with the extent of price discrimination. This result is particularly relevant to the analysis of mergers in antitrust, for which considerations of marginal efficiency (output restriction) and consumer equity (average prices paid) are relevant, and which are often analysed within the standard single-price oligopoly model framework, even when – as in the case of airlines and many other industries – it is clear in

fact that price discrimination is taking place.(HAZLEDINE, 2006)

Kutlu (2009) and Bakó e Kálecz-Simon (2012) confirm the robustness of the invariance of average price to level of price discrimination. Kutlu examines Hazledine (2006) Cournot game under a more general inverse demand function. He showed that the equilibrium quantity weighted average price is the same as the uniform price for a family of inverse demand functions which nests the linear functional form.

In its turn, Bakó e Kálecz-Simon (2012) analyzes third degree price discrimination as modeled by Hazledine (2006), but instead of using a symmetric setup they introduce asymmetric firms.Their main finding is that in any asymmetric Cournot oligopoly the output-weighted average price is not dependent on the extent of price discrimination, hence the standard single-price Cournot model does not mislead in this respect.

2.4 Robinson's Theory

On the moral of third-degree price discrimination, Robinson (1933) claims that discrimination must be held to be superior to simple monopoly in all these cases in which it leads to an increase of output. Perhaps because of this, the literature on the social superiority of price discrimination has generally focused on the comparison of outputs under simple and discriminatin monopolies.

Robinson also shows geometrically that if a single-price monopoly selling in two markets under constant costs is allowed to discriminate between them, total output is unchanged if both markets have linear demand curves. It follows at once that if the demand curves are straight lines, so that they ·coincide with their tangents, discriminating monopoly output is equal to simple monopoly output.

Since the total output is the same under discrimination when the demand curves are straight lines as it is under simple monopoly, it makes no difference whether marginal costs are rising, falling, o:r constant; the total output and the marginal cost are unchanged by the introduction of discrimination, though the prices, the outputs in the individual markets, and the profit of the monopolist are altered.(ROBINSON, 1933)

If demand curves are not linear, she argues that a comparison of their "adjusted concavities" at the nondiscriminating monopoly price determines whether total output rises or falls. It follows that when the more elastic demand curve is concave the increase in output in the separate market that results from discrimination will be greater than when the demand curve is a straight line. And when the less elastic curve is concave the decrease in output will be greater than for a straight line. (ROBINSON, 1933)

For any fixed total output of the monopolized product, efficiency requires that all buyers have the same marginal valuation of additional units. For example, if all buyers

are households, they must have the same marginal rate of substitution between the good involved and any numeraire good. Selling the same product at different prices to different buyers induces different marginal valuations and produces what Robinson (1933) terms "a maldistribution of resources as between different uses." Only an increase in total output above the single-price monopoly level can serve to offset this distributional inefficiency. Thus, unless total output increases, monopolistic third degree price discrimination produces a net efficiency loss.(SCHMALENSEE et al., 1980)

In sum, given that firms are equally efficient at production and aggregate output is unchanged, price discrimination reduces welfare because it generates interconsumer misallocations. More generally, when demand curves are non-linear or some markets would not be served under uniform pricing, price discrimination may increase welfare.(STOLE, 2007)

Robinson (1933) makes this point in the context of monopoly, but the economic logic applies more generally to imperfect competition, providing that the firms are equally efficient at production and the number of firms is fixed.(STOLE, 2007)

Borenstein (1985) and Holmes (1989) extend the analysis of third-degree price discrimination to settings of imperfect competition with product differentiation, underscoring the significance of cross-price elasticities in predicting changes in profits and surplus. In particular, Holmes (1989) builds upon the monopoly model of Robinson (1933) and demonstrates that under symmetric duopoly, it is crucial to know the ratio of market to cross-price elasticities, aside from the adjusted concavities of demand. The curvatures of the market demand curves are insufficient, by themselves, to predict changes in aggregate output when markets are imperfectly competitive.

For Cournot oligopoly, the adjusted concavities are key to determine the effect of price discrimination on aggregate output. When demand curves are linear, these concavities are zero, and – providing all market segments are served – price discrimination has no effect on total sales.(STOLE, 2007)

2.5 Price discrimination for Cournot in a two-stage game

Kumar e Kutlu (2016) differs from the earlier works with homogenous goods framework in that they analyze price discrimination as a two-stage game. In the first stage, the firms compete on quantities that they put in the market and in the second stage they simultaneously decide what fraction of the quantity they sell to different group of buyers. In other words, in the second stage firms optimally segment their customers by allocating the available quantities.

Forgiven capacities, Kumar and Kutlu provide an algorithm that defines the price

discrimination behaviour of the firms in second stage. The firms focus more on the high valuation customers which results in a coarse partition of high valuation customers and fine partition of low valuation customers. They solve the equilibrium price discrimination for Cournot as a benchmark case of first stage. It is found that the quantity weighted average price increases with the level of price discrimination asopposed to established results in the literature for one-stage games. Moreover, the deadweight loss does not vanish with the level of price discrimination as is the case with one-stage games and monopoly.

2.6 Empirical works

Empirically, the existence of third-degree price discrimination is well documented for a variety of products, for example, fresh fish by Graddy (1995) and mortgage lending by Ladd (1998). The studies of European car markets by Verboven (1996) and Goldberg e Verboven (2001) show that part of the cross-country price differentials are in fact due to third-degree price discrimination.

However, these studies do not directly relate third-degree price discrimination to market structure. There are a number of studies that analyze prices in the U.S. domestic flights markets. In a seminal paper, Borenstein e Rose (1994) find that when the number of competitors in a market grows, the observed price dispersion increases. The prices they analyze are a mix of a menu of fare classes available to everyone and special fares offered to exclusive groups. Thus, both second and third-degree price discrimination are observed and impossible to untangle.

3 Theoretical Framework

Firms may have many instruments at their disposal for discrimination between buyers. A firm can consider the valuations of consumers to be represented by a function of age, firms may have discounts for senior citizens and students. Another example is that of airline industry where the valuations of the buyers can be represented by a function of the time when they buy tickets. The business travellers whose plans are generally last moment have less elastic demand whereas the tourists whose plans are almost always flexible. Thus different bins of buyers can be grouped based on the day they want to buy a particular airline seat.(KUMAR; KUTLU, 2016)

Supposing that there is an industry composed of two firms, i and j, with the product of i differentiated from the product of j. Each firm produces at a constant marginal cost equal to $MC_i = MC_j = c$. The set of potential consumers of these products is exogenously partitioned into two groups, female and male. Also, let's consider that men have have the lower elastic demand when compared with women.

The firms can practice third-degree price discrimination which means it is feasible for firms to set one price in the female market and another in the male market. Let's assume that each firm sells in both market.

In addition, we will consider that the quantity sold to a certain group influences the inverse demand of the other. In other words, we will assume that there is a kind of interdependency between the types of consumers.

Considering a simple quasilinear model of two goods, let's assume that there are two commodities: the good in question, which in our case would be the tickets sold to women and men, and the numeraire good. The utility of a representative consumer is:

$$U^h(Q^h, Q^m) = m_h + a_h Q^h - \frac{1}{2}((Q^h)^2 - 2bQ^m Q^h) \tag{3.1}$$

$$U^m(Q^m) = m_m + a_m Q^m - \frac{1}{2}(Q^m)^2 \tag{3.2}$$

where q^h is the number of male consumers, q^m is the amount of female consumers, $Q^h = q_1^h + q_2^h + ... + q_n^h$,e $Q^m = q_1^m + q_2^m + ... + q_n^m$. m_h is the consumption of a numeraire good by men, m_m is the consumption of a numeraire good by women, and b is the externality parameter, $0 \leq b \leq 1$, because the two markets are complementary.

Here b is a constant that exhibits an effect of interdependency between the groups of consumer. When $b = 0$, quantity sold to female consumers does not influence inverse demand of male public. If $b = 1$, externality between the markets is maximum.

In view of the above, assuming that there are two price categories, p_h (men) e p_m (women), and that $Umg = p$ by the theory of partial competitive equilibrium, we have that the inverse demand is given by:

$$p_h(Q^h, Q^m) = a_h - Q^h + bq^m \tag{3.3}$$

$$p_m(Q^m) = a_m - Q^m \tag{3.4}$$

where a_h and a_m are the reservation price for each group, and $a_m, a_h \geq c_i$ (marginal cost).

3.1 Market with two firms

For the case of duopoly,the equations (3.3) e (3.4) can be expanded:

$$p_h(q_i^h, q_i^m, q_j^h, q_j^m) = a_h - q_i^h - q_j^h + bq_i^m + bq_j^m \tag{3.5}$$

$$p_m(q_i^m, q_j^m) = a_m - q_i^m - q_j^m \tag{3.6}$$

Now, we will consider the problem of profit maximization. Extending Cournot's duopoly model to the case of price discrimination, we have that i's profit is given by:

$$\Pi_i = (p_h - c)q_i^h + (p^m - c)q_i^m$$

Replacing 3.5 and 3.6 in profit's function:

$$\Pi_i = (a_h - q_i^h - q_j^h + bq_i^m + bq_j^m - c)q_i^h + (a_m - q_i^m - q_j^m - c)q_i^m \tag{3.7}$$

We solve the first-order conditions simultaneously for output and plug these optimal values into the demand and profit equations to obtain the Cournot equilibrium. In this case, the first-order conditions are:

$$\frac{\partial \Pi_i}{\partial q_i^h} = a_h - 2q_i^h - q_j^h + bq_i^m + bq_j^m - c = 0 \tag{3.8}$$

$$\frac{\partial \Pi_i}{\partial q_i^m} = bq_i^h + a_m - 2q_i^m - q_j^m - c = 0 \tag{3.9}$$

We want to find the Nash equilibrium values for the two different demands. Thus, rearranging (3.8) and (3.9) and isolating the variables q_i^h and q_i^m:

$$q_i^h = \frac{a_h - q_j^h + b(q_i^m + q_j^m) - c}{2} \tag{3.10}$$

$$q_i^m = \frac{bq_i^h + a_m - q_j^m - c}{2} \tag{3.11}$$

Replacing (3.10) in (3.11):

$$q_i^h = \frac{ba_h + 2a_m - bq_j^h + (b^2 - 2)q_j^m - (b + 2)c}{4 - b^2} \tag{3.12}$$

Therefore,

$$q_i^h = \frac{2a_h + ba_m - 2q_j^h + bq_j^m - (2 + b)c}{4 - b^2} \tag{3.13}$$

Now, let's find a symmetrical Nash equilibrium imposing $q_i^h = q_j^h = q^h$ and $q_i^m = q_j^m = q^m$, first order conditions are simplified to:

$$q^h = \frac{a_h + 2bq^m - c}{3} \tag{3.14}$$

In addition,

$$q^m = \frac{a_m + 2bq^h - c}{3} \tag{3.15}$$

Replacing (3.14) in (3.15):

$$q^m = \frac{3a_m + ba_h - (3 + b)c}{9 - 2b^2} \tag{3.16}$$

Thereby:

$$q^h = \frac{3a_h + 2ba_m - (3 + 2b)c}{9 - 2b^2} \tag{3.17}$$

So, we can conclude that:

Proposition 3.1.1 *Considering that all firms in the market can practice third-degree price discrimination, and the quantity sold to a certain group influences the inverse demand of the other through an exogenous parameter,b, which* $0 \leq b \leq 1$. *So, if* $a_h, a_m > c$, *the higher the value of parameter b, the greater the quantity sold to men and women. In other words, the greater the influence of quantity of women on the demand curve of men, the greater the quantity sold for both segments in a duopoly market.*

Proof:

Calculating the derivate with respect to externality variable, b:

$$\frac{\partial q^m}{\partial b} = \frac{12ba_m - 9a_h + 2b^2a_h - 9c - 2b^2c - 12bc}{(9 - 2b^2)^2} \tag{3.18}$$

Assuming that $a_h, a_m > c$, we have $\frac{\partial q^m}{\partial b} > 0$. Similarly,

$$\frac{\partial q^h}{\partial b} = \frac{4b^2a_m + 18a_m + 12a_hb - 18c - 4b^2c - 12bc}{(9 - 2b^2)^2} > 0 \tag{3.19}$$

These results were expected, because, the higher influence of women over men makes that the ticket for women to be cheaper to increase the amount of women in the bar/firm/club which will increase the amount of men tickets sold. It encourages the adoption of third price discrimination by the firms.

3.2 General Case

In a framework with n firms, the inverse demand function and the firm's profit $i = 1, ..., n$ are, respectively:

$$p_h(Q^h, Q^m) = a_h - Q^h + bQ^m \tag{3.20}$$

$$p_m(Q^m) = a_m - Q^m \tag{3.21}$$

where: $Q^h = q_1^h + q_2^h + ... + q_n^h = q_i^h + q_{-i}^h$ e $q_{-i}^h = \sum_{j \neq i} q_j^h$, $Q^m = q_1^m + q_2^m + ... + q_n^m = q_i^m + q_{-i}^m$ e $q_{-i}^m = \sum_{j \neq i} q_j^m$.

Let's consider the problem of profit maximization of firm i:

$$\Pi_i(q_1^h, ..., q_n^h, q_1^m, ..., q_n^m) = (a_h - Q^h + bQ^m - c)q_i^h + (a_m - Q^m - c)q_i^m \tag{3.22}$$

Rewriting the profit function:

$$\Pi_i(q_1^h, ..., q_n^h; q_1^m, ..., q_n^m) = (a_h - q_i^h - y_{-i}^h + b(q_i^m + y_{-i}^m) - c)q_i^h + (a_m - q_i^m - y_{-i}^m - c)q_i^m \tag{3.23}$$

Derivating with respect to q_i^h and q_i^m to find:

$$\frac{\partial \Pi_i}{\partial q_i^h} = a_h - 2q_i^h - y_{-i}^h + b(q_i^h + y_{-i}^m) - c = 0 \tag{3.24}$$

$$\frac{\partial \Pi_i}{\partial q_i^m} = a_m - 2q_i^m - y_{-i}^m + bq_i^h - c = 0 \tag{3.25}$$

We are looking for symetrical Nash equilibria, lets assume that $q_1^h = q_2^h = ... = q_n^h = q^h$ and $q_1^m = q_2^m = ... = q_n^m = q^m$.

Thus:

$$q_{-i}^h = \sum_{j=1}^{n} q_j^h - q_i^h = nq^h - q^h = (n-1)q^h$$

Similarly, $q_{-i}^m = (n-1)q^m$.

At that, we have:

$$q^h = \frac{a_h + bnq^m - c}{n+1} \tag{3.26}$$

$$q^m = \frac{a_m + bq^h - c}{n+1} \tag{3.27}$$

Making a substitution between the first and the second equation:

$$q^m = \frac{(n+1)a_m + ba_h - [(n+1)+b]c}{(n+1)^2 - b^2 n} \tag{3.28}$$

Consequently:

$$q^h = \frac{(n+1)a_h + bna_m - (n+1+bn)c}{(n+1)^2 - b^2 n} \tag{3.29}$$

We can reach the same conclusion as the previous section:

Proposition 3.2.1 *Considering that all firms in the market can practice third-degree price discrimination, and the quantity sold to a certain group influences the inverse demand of the other through an exogenous parameter,b, which* $0 \leq b \leq 1$. *So, if* $a_h, a_m > c$, *the higher the value of parameter b, the greater the quantity sold to men and women. In other words, the greater the influence of quantity of women on the demand curve of men, the greater the quantity sold for both segments in a market with n firms.*

Proof:

Calculating the derivate with respect to externality variable, b:

$$\frac{\partial q^m}{\partial b} = \frac{[(n+1)^2 + nb^2](a_h - c) + (n+1)(2nb)(a_m - c)}{[(n+1)^2 - b^2 n]^2} \tag{3.30}$$

Suposing that $a_h, a_m > c$, we have $\frac{\partial q^m}{\partial b} > 0$. Similarly,

$$\frac{\partial q^h}{\partial b} = \frac{[(n+1)^2 n + n^2 b^2](a_m - c) + (n+1)(2nb)(a_h - c)}{[(n+1)^2 - b^2 n]^2} > 0 \tag{3.31}$$

Proposition 3.2.2 *Considering that all firms in the market can practice third-degree price discrimination, and the quantity sold to a certain group influences the inverse demand of the other through an exogenous parameter,b, which* $0 \leq b \leq 1$. *So, if* $a_h, a_m > c$, *the greater the number of firms (n) in the market, the less the quantity sold to all consumer groups, because there is more competition and each firm can not sell the same amount of tickets if more firms open.*

Proof:

Now, derivating 3.28 and 3.29 with respect to n, we obtain:

$$\frac{\partial q^m}{\partial n} = \frac{[(n+1)^2 - b^2](c - a_m) + (2nb + 2b - b^3)(c - a_h)}{[(n+1)^2 - b^2 n]^2} \tag{3.32}$$

As $a_h, a_m > c$, we have $\frac{\partial q^m}{\partial n} < 0$. As with the other public,

$$\frac{\partial q^h}{\partial n} = \frac{[(n+1)^2 n - b^2](c - a_h) + (n^2 - 1)(c - a_m)}{[(n+1)^2 - b^2 n]^2} < 0 \tag{3.33}$$

3.3 Total Output

The total output of the female public is:

$$Q^m = n\left[\frac{(n+1)a_m + ba_h - (n+1+b)c}{(n+1)^2 - b^2 n}\right] \tag{3.34}$$

When n tends to infinity, we have:

$$\lim_{n\to\infty} Q^m = a_m + ba_h - (1+b)c \tag{3.35}$$

Analogously, for male consumers:

$$Q^h = n\left[\frac{(n+1)a_h + bna_m - (n+1+bn)c}{(n+1)^2 - b^2 n}\right] \tag{3.36}$$

$$\lim_{n\to\infty} Q^h = a_h + ba_m - (1+b)c \tag{3.37}$$

For quantity sold to women to be greater than the quantity sold to men as the expected result, it is necessary:

$$Q^m \geq Q^h \Leftrightarrow a_m - a_h \geq b(a_m - c) \tag{3.38}$$

As $a_m > c$ and $b > 0$, we have that $a_m > a_h$. Therefore, the women's reservation price needs to be greater than the resevation price of men.

Proposition 3.3.1 *Considering that all firms in the market can practice third-degree price discrimination, and the quantity sold to a certain group influences the inverse demand of the other through an exogenous factor,b, which* $0 \leq b \leq 1$*. So, if* $a_h, a_m > c$ *and if the market is segmented in two groups, the group, whose has higher reserve price, it buys a larger quantity than the other group.*

Calculating the price for each segment:

$$p^h = a_h - Q^h - bQ^m \tag{3.39}$$

$$p^m = a_m - Q^m \tag{3.40}$$

Once again, considering that n tends to infinity:

$$\lim_{n\to\infty} p^h = b^2 a_h + c(1 - b^2) \tag{3.41}$$

$$\lim_{n\to\infty} p^m = c(1 - b^2) - ba_h \tag{3.42}$$

Following condition is necessary for the price charged to the female audience to be lower as expected:

$$p^m \leq p^h \Leftrightarrow a_h \geq c \tag{3.43}$$

which is one of the model's hypotheses.

Thus,

Proposition 3.3.2 *Considering that all firms in the market can practice third-degree price discrimination, and the quantity sold to a certain group influences the inverse demand of the other through an exogenous factor,b, which* $0 \leq b \leq 1$*. So, if* $a_h, a_m > c$ *and if the market is segmented in two groups, a lower price is charged for the influencing group, whose the reservation price is higher.*

Besides, analyzing the results, we obtain the values of the traditional case of Cournot oligopoly when n tends to infinity and $b = 0$, which helps to corroborate the model proposed in this dissertation.

3.4 Price Elasticity of Demand

According to Varian e Varian (1992), the price elasticity of demand is defined to be the percentage change in quantity demanded, q, divided by the percentage change in price, p. The formula for the demand elasticity ϵ is:

$$\epsilon = \frac{p}{q}\frac{\partial q}{\partial p} \tag{3.44}$$

Note that the law of demand implies that $\frac{\partial q}{\partial p} < 0$, and so ϵ will be a negative number. In some contexts, it is common to introduce a minus sign in this formula to make this quantity positive. You should be careful in all circumstances to check which definition is being used.

One way to see why price elasticity of demand might be useful is to consider the question: How will revenue change as we change price?

Recall that revenue is price times quantity demanded. If we write everything in terms of price (by using the demand equation $q = q(p)$), we get:

$$R(p) = pq(p) \tag{3.45}$$

Now, the derivative of a function tells us how that function will change: If $\frac{\partial R(p)}{\partial p} > 0$ then revenue is increasing at that price point, and $\frac{\partial R(p)}{\partial p} < 0$ would say that revenue is decreasing at that price point. So, we compute $\frac{\partial R(p)}{\partial p}$ using the general revenue equation:

$$\frac{\partial R(p)}{\partial p} = q(p) + p\frac{\partial q}{\partial p}$$

$$\frac{\partial R(p)}{\partial p} = q(p)[1 + \frac{p}{q(p)}\frac{\partial q}{\partial p}] \tag{3.46}$$

Note that $q(p)$ is always positive, so the sign of $\frac{\partial R(p)}{\partial p}(p)$ is completely determine by the quantity in the brackets. We note that whether this quantity is positive or negative is determined by the size of the second term, $\frac{p}{q(p)}\frac{\partial q}{\partial p}$, which is precisely the demand elasticity ϵ as we have defined it. Thus, the size, in absolute value, of ϵ relative to 1 will determine how the revenue changes as the price changes.

Now, suppose a manager wants to find out how consumers will react when he increases the price of a good. If increasing the price increases revenue, the manager would have made a good decision. If increasing the price decreases revenue, the manger would

have made a bad decision. We use elasticities to see how sensitive are consumers to price changes.

Remember, the negative value of ϱ comes from the factor $\frac{\partial q}{\partial p}$, which is the slope of the demand curve which is negative by the law of demand. Economists use the following terminolgy:

1. when $|\epsilon| > 1$, we say the good is price elastic. In this case, management should decrease price to have a higher revenue.
2. when $|\epsilon| < 1$, we say the good is price inelastic. In this case, management should increase price to have a higher revenue.
3. when $|\epsilon| = 1$, we say the good is price unit elastic.

3.4.1 Application to discriminate market

Remembering that, in a game with n firms, the inverse demand function of male and female market are, respectively:

$$p_h(q_1^h, ..., q_n^h, q_1^m, ..., q_n^m) = a_h - Q_h + bQ_m \tag{3.47}$$

$$p_m(q_1^m, ..., q_n^m) = a_m - Q_m \tag{3.48}$$

Let's consider the problem of measuring the demand elasticity. So, according to the last section, we have:

$$\epsilon = \frac{p}{q(p)}\frac{\partial q}{\partial p} \tag{3.49}$$

In our case, the strategy of the firms is symmetrical and, consequently, the demand function, $q(p)$, is similar. So, we will regard the aggregate demand function, $Q(p)$, which is given by:

$$Q^h(p_h) = a_h + bQ^m - p_h \tag{3.50}$$

$$Q^m(p_m) = a_m - p_m \tag{3.51}$$

Calculing the the demand elasticity in both market:

$$\epsilon_h = \frac{p_h}{Q^h(p_h)}\frac{\partial Q^h}{\partial p_h} \longrightarrow \epsilon_h = \frac{p_h}{a_h + bQ^m - p_h}(-1) \tag{3.52}$$

$$\epsilon_m = \frac{p_m}{Q^m(p_m)}\frac{\partial Q^m}{\partial p_m} \longrightarrow \epsilon_m = \frac{p_m}{a_m - p_m}(-1) \tag{3.53}$$

In the strategy of third discrimination, a firm need to know what group of consumers have the lower elastic demand, because, at this group, management should increase price to have a higher revenue as we have seen in the previous section. At that, in our context, we assume that male market is this group with less elasticity. Thus:

$$\epsilon_m > \epsilon_h$$

$$\therefore Q^m > \frac{a_m - a_h}{b} \tag{3.54}$$

As the previous results obtained in this work, we know that $a_m > a_h$ and $0 < b < 1$. Furthermore, one of our basic assumptions is that each firm sells in both market discriminated. So, necessarily, $Q^m > 0$, because each firm needs to sell, at least, one product for a female consumer. It therefore has to be concluded that men have a lower elastic demand than women.

4 Welfare Analysis

According to Aguirre, Cowan e Vickers (2010), the effect of discrimination on welfare can be divided into a misallocation effect and an output effect. With discrimination output is inefficiently distributed because consumers face different prices in different markets. This negative feature of discrimination may, however, be offset if there is an increase in total output, which is socially valuable since prices exceed marginal costs. Thus, in welfare evaluation, these effects should not be ignored.

Since the seminal work of Robinson (1933), the main focus of the analysis of third-degree price discrimination has been on changes in social welfare. One of the most important findings is stated by Schmalensee et al. (1980): that an increase in total output by price discrimination is a necessary condition for aggregate social welfare (i.e. the sum of the monopolist's profit and consumers' surplus) to increase. Moreover, in the linear demand case, (a) total output does not change by price discrimination, so that (b) the social welfare necessarily decreases by price discrimination.

Borenstein e Rose (1994) and Holmes (1989) extend the analysis of third-degree price discrimination to settings of imperfect competition with product differentiation, underscoring the significance of cross-price elasticities in predicting changes in profits and surplus. In particular, Holmes (1989) builds upon the monopoly model of Robinson (1933) and demonstrates that under symmetric duopoly, it is crucial to know the ratio of market to cross-price elasticities, aside from the adjusted concavities of demand. The curvatures of the market demand curves are insufficient, by themselves, to predict changes in aggregate output when markets are imperfectly competitive.

In this dissertation, we try to extend the traditional argument of oligopolistic third-degree price discrimination for a single final product in an economic environment where consumers' tastes are exhibiting consumption externalities within two separate markets.

Generally speaking, third-degree price discrimination makes the amounts of consumption in each group larger or smaller than when firms are engaged in uniform pricing. Thus, it necessarily changes the ratio of supply to particular groups, even if the amount of aggregate consumption or total number of consumers who really purchase the good remains unchanged. It is reasonable to hold that these shifts would noticeably affect the willingness-topay of all customers in the presence of consumption externalities. So, it is very important analysis welfare's behavior.

So, in this chapter, we will compare the welfare under discriminate price and under uniform price to conclude whether or not the practice of third-degree price discrimination

by oligopolists in Cournot model is better. In order to get this, we will firt calculate consumer surplus and firm's profit by the traditional way. After that, we will apply Holmes (1989) analysis in an oligopolistic market with an externality parameter.

4.1 Consumer and Producer Suplus

According to Varian e Varian (1992), the classical tool for measuring welfare changes is consumer's suplus and firm's profit. To calculate consumer's surplus (CS), we first need to remember aggregate utility function of each market:

$$U_h(Q^h, Q^m) = a_h Q^h - \frac{1}{2}((Q^h)^2 - 2bQ^m Q^h) \tag{4.1}$$

$$U_m(Q^m) = a_m Q^m - \frac{1}{2}(Q^m)^2 \tag{4.2}$$

As $p_j = Umg_j$, where$j = m, h$, we obtain to n firms, which are in symmetrical equilibria:

$$p_h(Q^h, Q^m) = a_h - Q^h + bQ^m \tag{4.3}$$

$$p_m(Q^m) = a_m - Q^m \tag{4.4}$$

Considering that consumer surplus is given by:

$$CS_j = U_j - p_j Q^j \tag{4.5}$$

We have:

$$CS_h(Q^h) = \frac{(Q^h)^2}{2} \tag{4.6}$$

$$CS_m(Q^m) = \frac{(Q^m)^2}{2} \tag{4.7}$$

So, we can conclude that:

$$CS(Q^h, Q^m) = \frac{(Q^h)^2}{2} + \frac{(Q^m)^2}{2} \tag{4.8}$$

With regard to producer surplus (PS), it is the sum over all units of production of the difference between the market price and the marginal cost of production.

So, producer surplus is related to profit, but is not equal to it. Producer's surplus subtracts only variable costs from revenues, while profit subtracts both variable and fixed costs. Consequently, producer's surplus is always greater than profit.

However, in our case, fixed cost is zero. Thus, we can consider that:

$$PS(q^h, q^m) = \sum_{i=1}^{n} \Pi_i(^h_i, q_i^m) \tag{4.9}$$

4.2 Under uniform pricing

With uniform pricing, both markets must have the same price. Thus:

$$p_h(Q_h, Q_m) = a_h - Q_h + bQ_m \tag{4.10}$$

$$p_m(Q_m) = a_m Q_m \tag{4.11}$$

Then, firm i maximizes:

$$\begin{aligned} \underset{q_i^h,\, q_i^m}{\text{maximize}} \quad & \Pi_i = (a_h - Q_h + bQ_m)q_i^h + (a_m - Q_m - c)q_i^m \\ \text{subject to} \quad & a_h - a_m - Q_h + Q_m(1+b) = 0 \end{aligned} \tag{4.12}$$

where $Q_h = q_i^h + y_{-i}^h$, $Q_m = q_i^m + y_{-i}^m$.

Rewriting the otimization problem:

$$\begin{aligned} \underset{q_i^h,\, q_i^m}{\text{maximize}} \quad & \Pi_i = (a_h - q_i^h - y_{-i}^h + bq_i^m + by_{-i}^m)q_i^h + (a_m - q_i^m - y_{-i}^m - c)q_i^m \\ \text{subject to} \quad & a_h - a_m - q_i^h - y_{-i}^h + (q_i^m + y_{-i}^m)(1+b) = 0 \end{aligned} \tag{4.13}$$

We define the Lagrangian as:

$$L = (a_h - q_i^h - y_{-i}^h + bq_i^m + by_{-i}^m)q_i^h + (a_m - q_i^m - y_{-i}^m - c)q_i^m + \lambda(a_h - a_m - q_i^h - y_{-i}^h + (q_i^m + y_{-i}^m)(1+b)) \tag{4.14}$$

Differentiating with respect to q_i^h, q_i^m and λ:

$$\frac{\partial L}{\partial q_i^h} = a_h - 2q_i^h - y_{-i}^h + bq_i^m + by_{-i}^m - c - \lambda = 0 \tag{4.15}$$

$$\frac{\partial L}{\partial q_i^m} = bq_i^h + a_m - 2q_i^m - y_{-i}^m - c + \lambda(1+b) = 0 \tag{4.16}$$

$$\frac{\partial L}{\partial \lambda} = a_h - a_m - q_i^h - y_{-i}^h + (q_i^m + y_{-i}^m)(1+b) = 0 \tag{4.17}$$

As the strategy of the firms is symmetrical, $y_{-i}^h = (n-1)q^h$ and $y_{-i}^m = (n-1)q^m$, and applying addition method:

$$q^m = \frac{(nb+2n+1)a_m - a_h - cn(2+b)}{2n^2+2n+n^2b+nb} \tag{4.18}$$

$$q^h = \frac{(nb+2n+1)a_h + (2nb+nb^2-1)a_m - cn(1+b)(2+b)}{2n^2+2n+n^2b+nb} \tag{4.19}$$

Computing total production in female market:

$$Q^m = \frac{(nb+2n+1)a_m - a_h - cn(2+b)}{2n+2+nb+b} \tag{4.20}$$

When n tends to infinity:

$$\lim_{n\to\infty} Q^m = a_m - c \tag{4.21}$$

which is the same result obtained under price discrimination.

In male market:

$$Q^h = \frac{(nb+2n+1)a_h + (2nb+nb^2-1)a_m - cn(1+b)(2+b)}{2n+2+nb+b} \tag{4.22}$$

$$\lim_{n\to\infty} Q^h = a_h - c \tag{4.23}$$

which is also the same result obtained under price discrimination if $b = 0$.

4.3 Comparison of social welfare between uniform and discriminate regimes

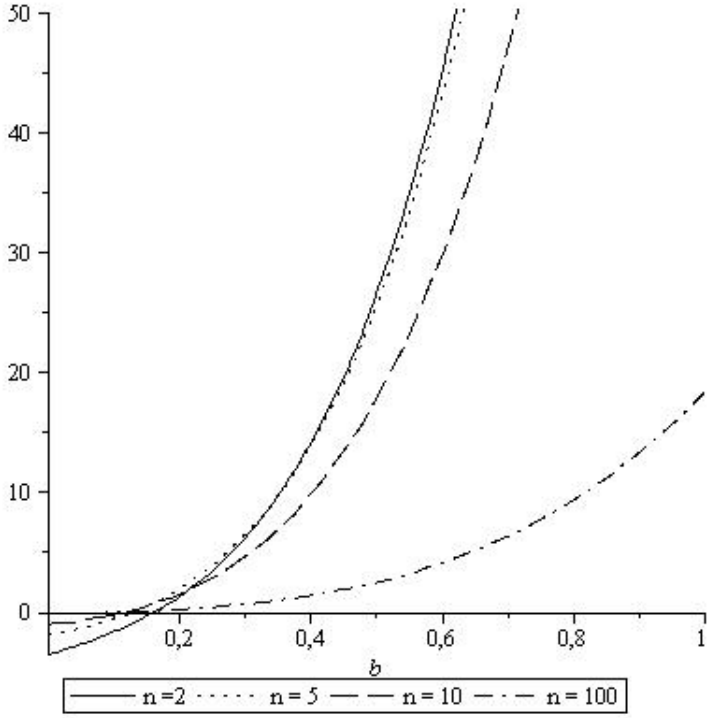
As it is difficult to directly compare social welfare in the two regimes, uniform and discriminate, we will stablish values for some parameters of our problem.So, knowing that $a_m \geq a_h$ and $c \leq a_h$, we assume $a_m = 30, a_h = 25$ and $c = 6$.

As we want to know if discriminated price is better than uniforming one, we will compute the difference between total consumer surplus in the regimes, in other words, we will examine:

$$\Delta CS = CS_{PD} - CS_U \tag{4.24}$$

Graphically, we get:

Figure 4 – ΔCS as a function of b



Analysing Figure 4, we observe that, for values of $b > 0.2$, price discrimination is better than uniform pricing for consumers. It means that, an increased dependence between women and men function creates more favourable context to third price discrimination strategy. Also, we can note that the greater the number of firms, the smallest difference between the consumer surplus derived from each price rule.

So, considering that all firms in the market can practice third-degree price discrimination, and the quantity sold to a certain group influences the inverse demand of the other through an exogenous factor,b, which $0 \leq b \leq 1$. So, for values of b close one[1], it is more advantageous for consumers to participate of a price discriminate market than of an uniform one. At the same time,the higher the number of firms, smaller is the diffence between the consumer surplus through under price discrimination and under uniform pricing. That is, consumers care less about the price strategy adopted by the firms.

A further question that needs to be addressed is the gain of female consumers. To verify if women have welfare gains in discriminated market, we will, separately, display two graphs, one to female consumer surplus and another to male consumer surplus in both price strategy. First, we will consider $n = 2$.

Figure 5 shows that, for values of $b > 0.16$, third price discrimination is always better than uniform pricing to female consumers. So, high values of interdependency

[1] Maybe we can achieve algebraically this result through Mean Value Theorem (MVT)

between the groups of consumers increase the consumer surplus of women in discriminated market when compared with uniform market.

Figure 5 – Women's consumer surplus as a function of b with $n = 2$

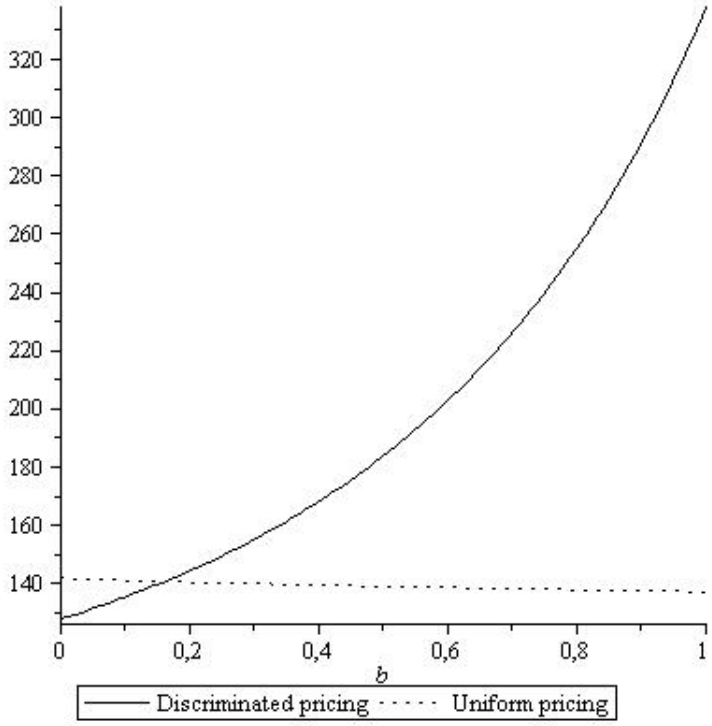
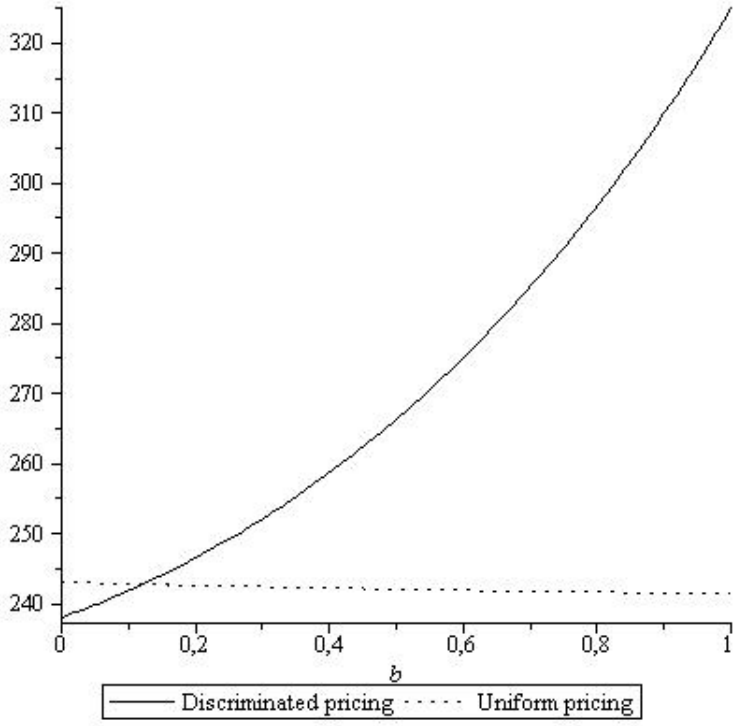


Figure 6 – Women's consumer surplus as a function of b with $n = 10$

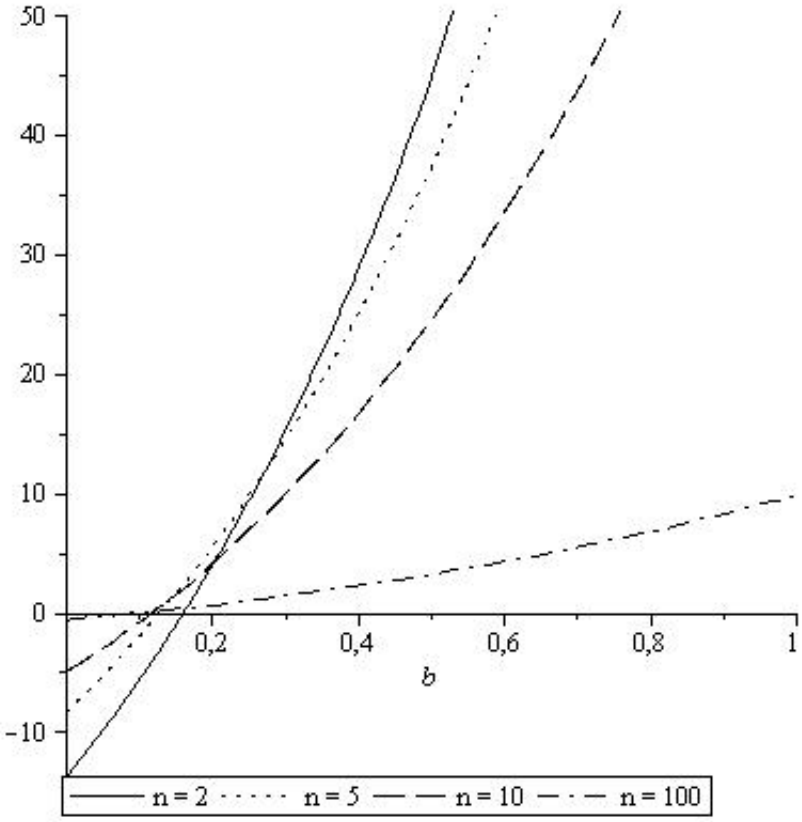


But, an interesting question could emerge when we think in the number of firms. Does the result remains the same in a market with $n = 10$? To answer this question, we need to analyse the Figure 6.

We note that the performance of women's consumer surplus still the same, but the values in a market with $n = 2$ is much bigger than with $n = 10$.So, the higher the number of firms, smaller is the diffence between the consumer surplus through under price discrimination and under uniform pricing.

To verify this conclusion, we will consider $\Delta CS^m = CS^m_{PD} - CS^m_U$, where ΔCS^h shows the difference between women's consumer surplus in a discriminated market and an uniform market. It is easy to note in Figure 7 that the difference between women's consumer surplus in the regimes is smaller as the number of firms increases, which confirms our guess.

Figure 7 – ΔCS^m as a function of b



Considering male consumers, we observe in Figure 8 that men's consumer surplus have some situations, where uniform pricing is better that discriminated one. So, the influencer market are those that benefit the most from third price discrimination.

In a market with $n = 10$, men's consumer surplus has almost the same performance in both regimes acordding to Figure 9, increasing the difference when b is closed to one.

It demonstrates that, male consumers care less about the price strategy adopted by the firms.

Figure 8 – Men's consumer surplus as a function of b with $n = 2$

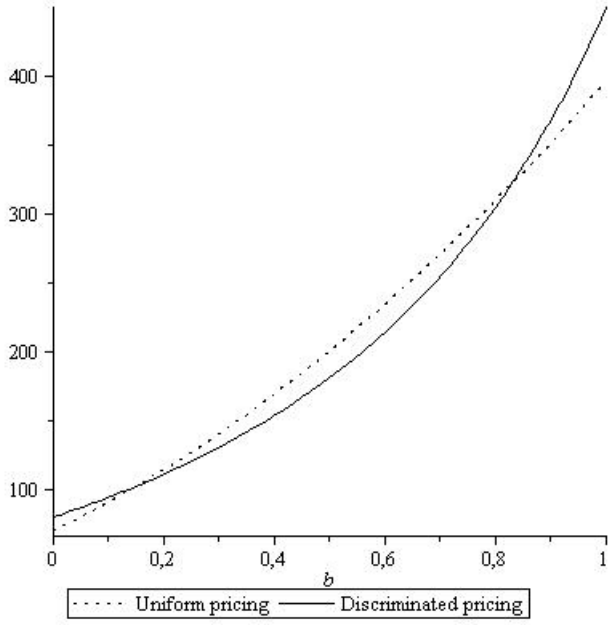
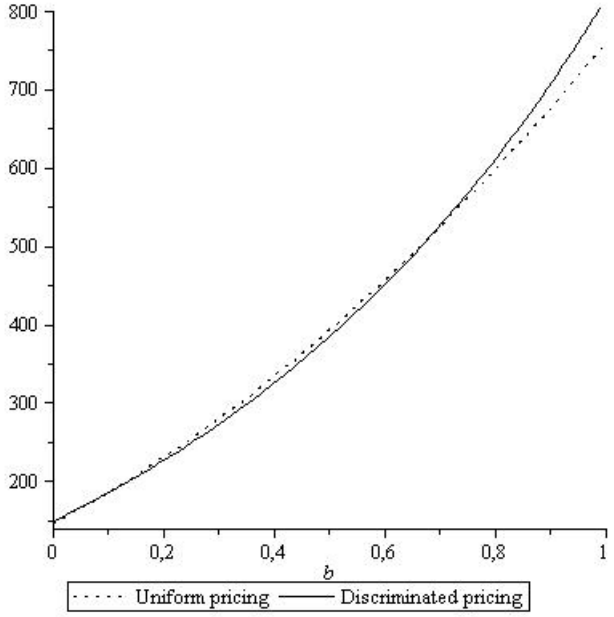


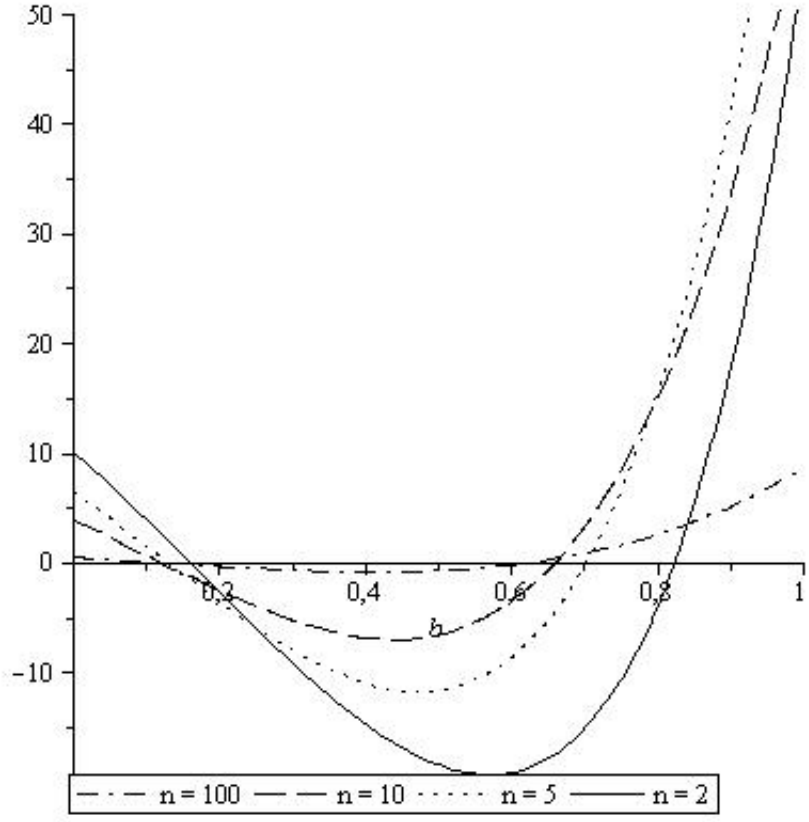
Figure 9 – Men's consumer surplus as a function of b with $n = 10$



Similarly, if we plot a graph based on $\Delta CS^h = CS^h_{PD} - CS^h_U$, where ΔCS^h shows the difference between men's consumer surplus in a price discriminated market and an uniform pricing market, we note that, for some values of b, men's consumer surplus is bigger with third price discrimination strategy. But, the higher the number of firms, smaller

is the diffence between the consumer surplus through under price discrimination and under uniform pricing.

Figure 10 – ΔCS^h as a function of b

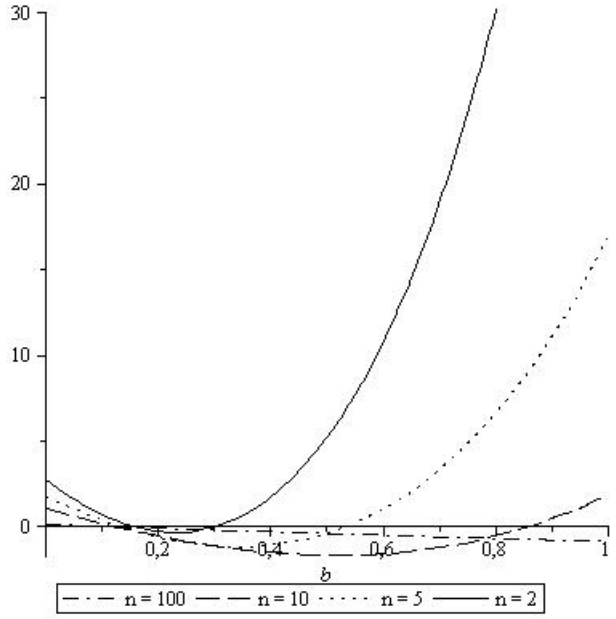


In the case of producer surplus, we need to calculate the difference between total producer surplus in the regimes:

$$\Delta PS = PS_{PD} - PS_U \tag{4.25}$$

Examing the Figure 11, we note similar behaviour when compared with male consumer surplus. Market under discriminated price results in a higher producer surplus than in a market under uniform pricing in some moments. For example, in duopolies, we observe that third price discrimination can obtain a better performance when $b < 0.16$ or $b > 0.3$. A similar situation occurs when $n = 10$, and $b < 0.12$ or $b > 0.86$. Furthermore,the higher the number of firms, smallest the difference between the price strategies.

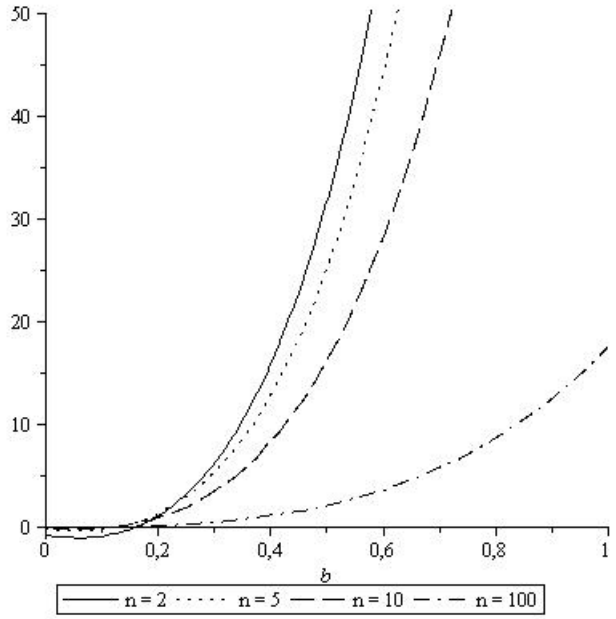
Figure 11 – ΔPS as a function of b



Therefore, considering that social welfare is the sum of consumer surplus and producer surplus, and, again calculing the difference between welfare obtained in the regimes: $\Delta SW = SW_{PD} - SW_U$.

Grafically, we obtain Figure 12, where we can note that social welfare under price discriminated is bigger than under uniform one, unlesse the case when b is closed to zero. But as n increases, this advantage tends to zero.

Figure 12 – ΔSW as a function of b



Conclusion

Price discrimination arises naturally in the theory of monopoly and oligopoly. Whenever a good is sold at a price in excess of its marginal cost, there is an incentive to engage in price discrimination. For to say that price is in excess of marginal cost is to say that there is someone who is willing to pay more than the cost of production for an extra unit of the good. Lowering the price to all consumers may well be unprofitable, but lowering the price to the marginal consumer alone will likely be profitable.(VARIAN, 1987)

In order to lower the price only to the marginal consumer, or more generally to some specific class of consumers, the firm must have a way to sort consumers. The easiest case is where the firm can explicitly sort consumers with respect to some exogenous category such as age. A more complex analysis is necessary when the firm must price discriminate on the basis of some endogenous category such as time of purchase. In this case the monopolist faces the problem of structuring his pricing so that consumers "self select" into appropriate categories.(VARIAN, 1987)

When the firm is a monopolist with market power, the underlying theory of price discrimination is now well understood, as explained, for example, by Varian (1987). On the other extreme, when markets are perfectly competitive and firms have neither short-run nor long-run market power, the law of one price prevails and price discrimination cannot exist.2 Economic reality, of course, largely lies somewhere in between the textbook extremes, and most economists agree that price discrimination arises in oligopoly settings. (STOLE, 2007)

In the context of price discrimination in oligopoly market, we can find the situation of the female public in the market of night parties, where the practice of price discrimination by gender is recurrent. Furthermore, it is possible to notice that the female public has a clear influence on men's demand for ticket to a particular party. In those situations, where a consumer group influences the other's demand curve, a kind of externality arises over this function.

In such context, the present work sought to analyze the role of externality in third degree price discrimination strategy and its impact on social welfare. Also, we checked whether the prohibition of price discrimination increases women and men's social welfare through Cournot's oligopoly model.

In reply to the first question, we found that, the greater the quantity sold to men and women. In other words, the greater the influence of quantity of women on the demand curve of men, the greater the quantity sold for both segments in a market with n firms.

The same results are obtained in relation to the number of firms in the market.

If the market is segmented in two groups, the group, whose has higher reserve price, it buys a larger quantity than the other group. Furthermore, we concluded that a lower price is charged for the influencing group, which has higher reserve price. In our case, for example, the female consumers pay a lower price than the male public.

With regard to social welfare, we observe that, when values of b close to one, market under price discrimination results in a higher total consumer surplus than a market under uniform pricing. But, as n increases, this advantage tends to zero. Analysing women's consumer surplus, values of b close to one, it is more advantageous for female consumers to participate of a discriminate market than of an uniform one. However,the higher the number of firms, smaller is the gain of these consumers through price discrimination.

Men's consumer surplus is bigger with third price discrimination strategy for some values of b. Producer surplus have a similar performance. So, we can conclude that the third price discrimination is more advantageous for female consumers than for men and producers.

A point to be analysed in the model is the use of linear function. Future discussion could implement other types of mathematical function, like quadratic and logarithmic. That way, the results obtained by Robinson (1933) and improved by Schmalensee et al. (1980) can be verified in a context of price discrimination with interdependency between the types of consumers.

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